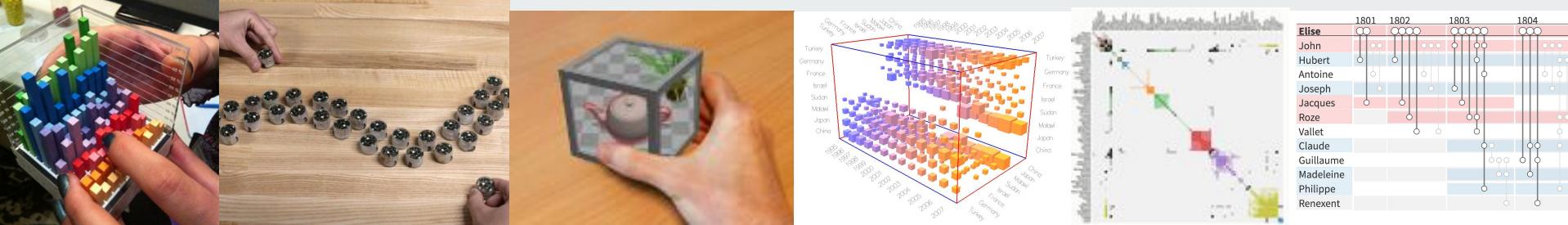


---

# Visualization of Graphs

Jean-Daniel Fekete, Inria  
<http://www.aviz.fr/~fekete>

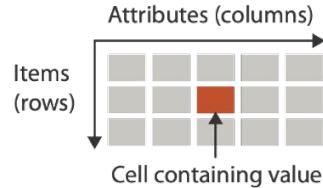


# Network data

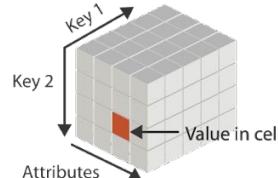
- networks
  - model relationships between things
    - aka graphs
  - two kinds of items, both can have attributes
    - nodes
    - links
- tree
  - special case
  - no cycles
    - one parent per node

## Dataset Types

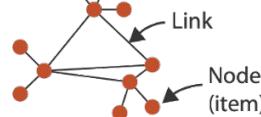
### → Tables



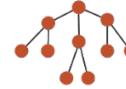
### → Multidimensional Table



### → Networks

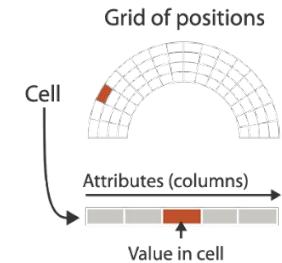


### → Trees



### → Spatial

### → Fields (Continuous)



# Formal definition

- A graph is:
  - A set of Vertices  $V=\{v_i\}$
  - A set of edges  $E=\{e_j\}$  with  $e=(v_s, v_d) \in V \times V$
  - When the order of the couple in  $E$  is meaningful, the graph is **directed**, otherwise, it is **undirected**
  - A graph is a mapping of  $V$  into  $V$
  - From there, we can define several measures or intrinsic properties on a graph

# Network tasks: topology-based and attribute-based

- topology based tasks

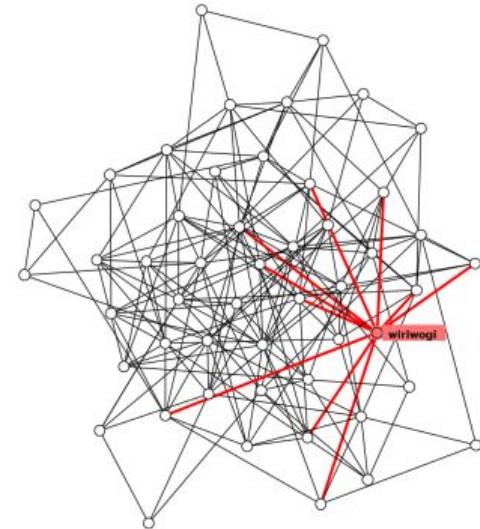
- find paths
  - find (topological) neighbors
  - compare centrality/importance measures
  - identify clusters / communities

- attribute based tasks (similar to table data)

- find distributions, ...

- combination tasks, incorporating both

- example: find friends-of-friends who like cats
    - topology: find all adjacent nodes of given node
    - attributes: check if has-pet (node attribute) == cat



# Issues

- Graph Layout
- Scalability
- Navigation

Beware of the hairball!

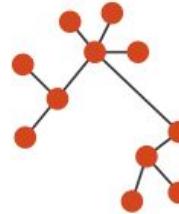
## Arrange Networks and Trees

### → Node–Link Diagrams

Connection Marks

NETWORKS

TREES

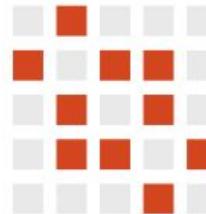


### → Adjacency Matrix

Derived Table

NETWORKS

TREES



### → Enclosure

Containment Marks

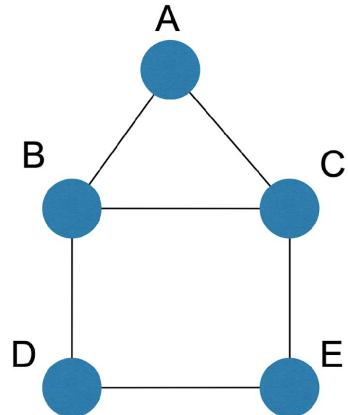
NETWORKS

TREES



# Node-link diagrams

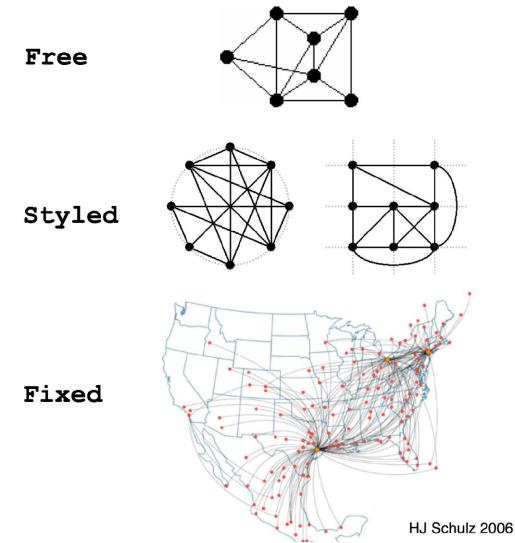
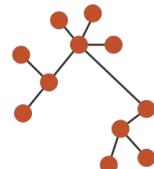
- nodes: point marks
- links: line marks
  - straight lines or arcs
  - connections between nodes
- intuitive & familiar
  - most common
  - many, many variants



## → Node–Link Diagrams

Connection Marks

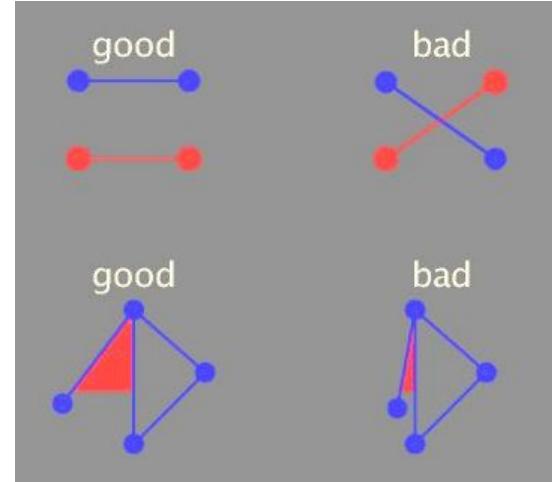
✓ NETWORKS ✓ TREES



HJ Schulz 2006

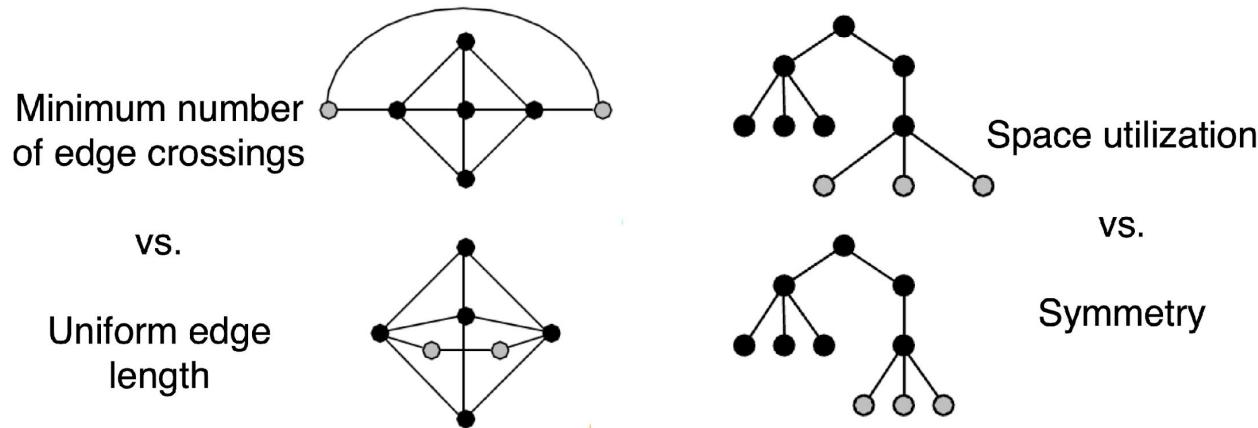
# Criteria for good node-link layouts

- minimize
  - edge crossings, node overlaps
  - distances between topological neighbor nodes
  - total drawing area
  - edge bends
- maximize
  - angular distance between different edges
  - aspect ratio disparities
- emphasize symmetry
  - similar graph structures should look similar in layout

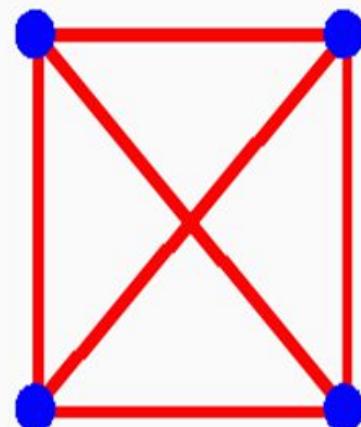
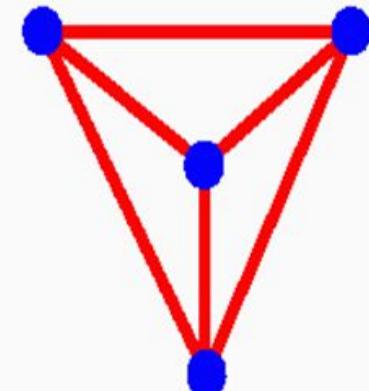
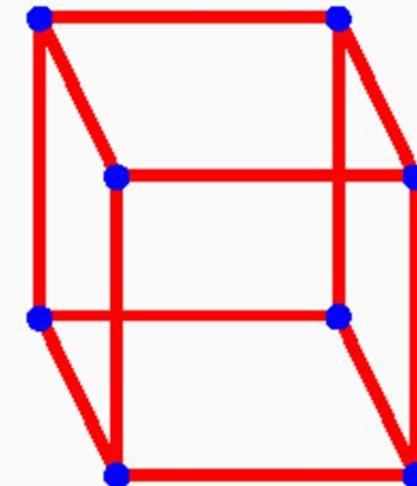
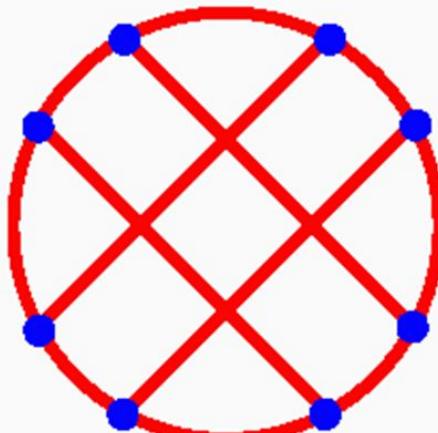
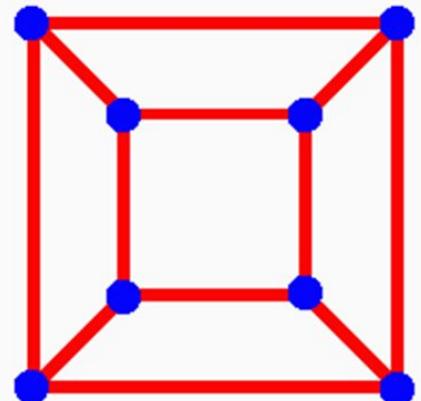


# Criteria conflict

- most criteria NP-hard individually
- many criteria directly conflict with each other



Schulz 2004



# Optimization-based layouts

- formulate layout problem as optimization problem
- convert criteria into weighted cost function
  - $F(\text{layout}) = a * [\text{crossing counts}] + b * [\text{drawing space used}] + \dots$
- use known optimization techniques to find layout at minimal cost
  - energy-based physics models
  - force-directed placement
  - spring embedders

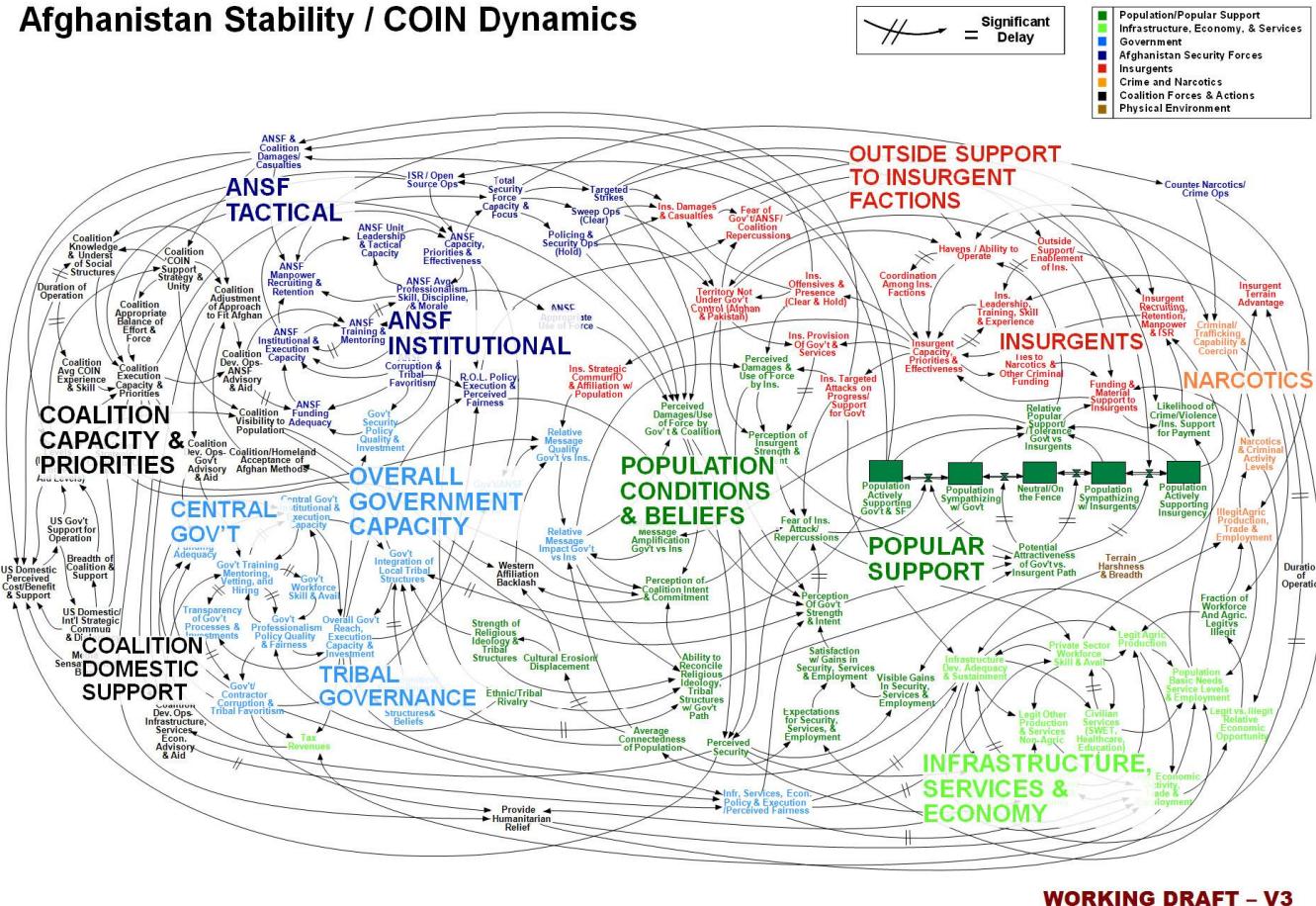


facebook

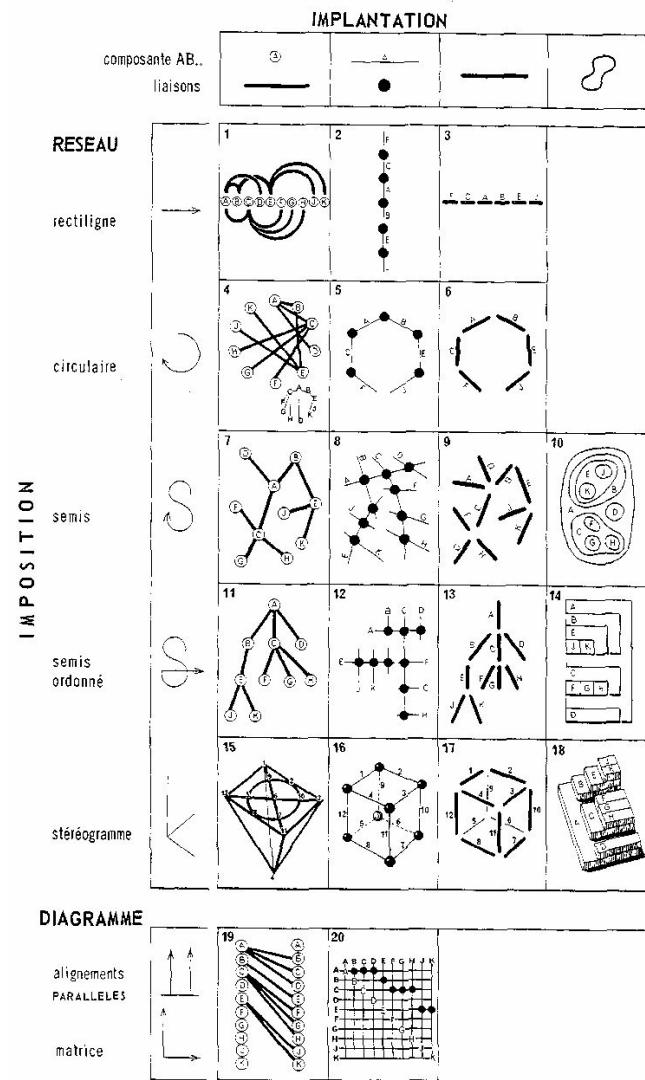
December 2010

# Afghanistan Stability / COIN Dynamics

Bummiler, 2010



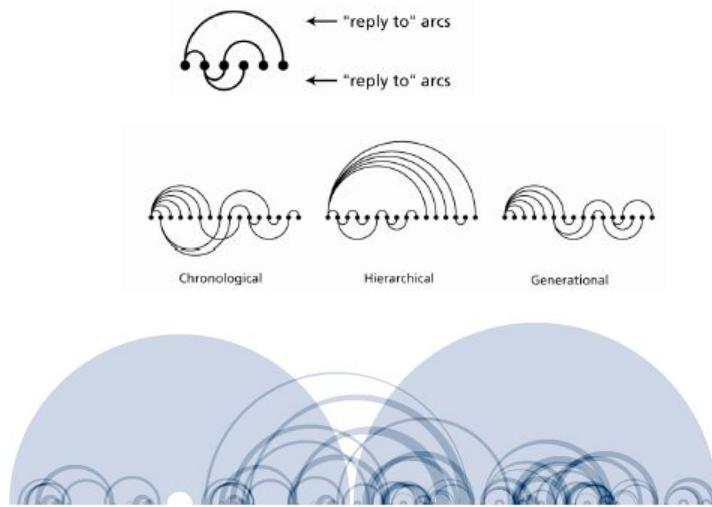
# Bertin 67





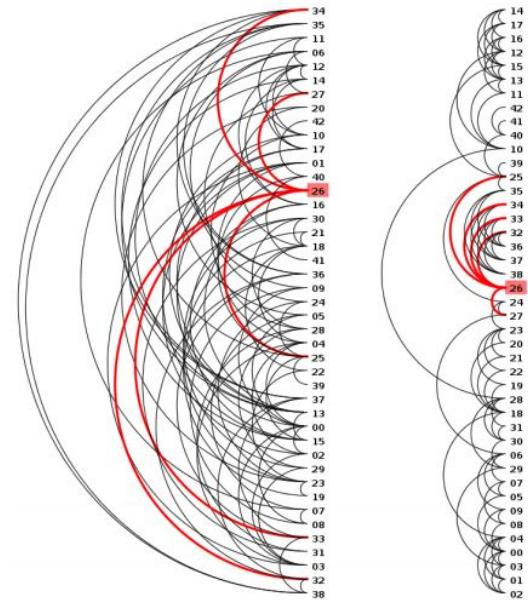
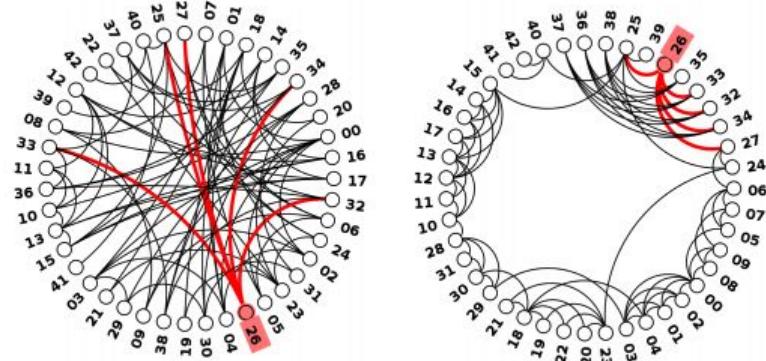
# Linear layout

- **THREAD ARCS: An Email Thread Visualization**  
(Bernard Kerr, IBM)
- Arc Diagrams: The Shape of Song (Martin Wattenberg, IBM)
- <http://www.bewitched.com/>



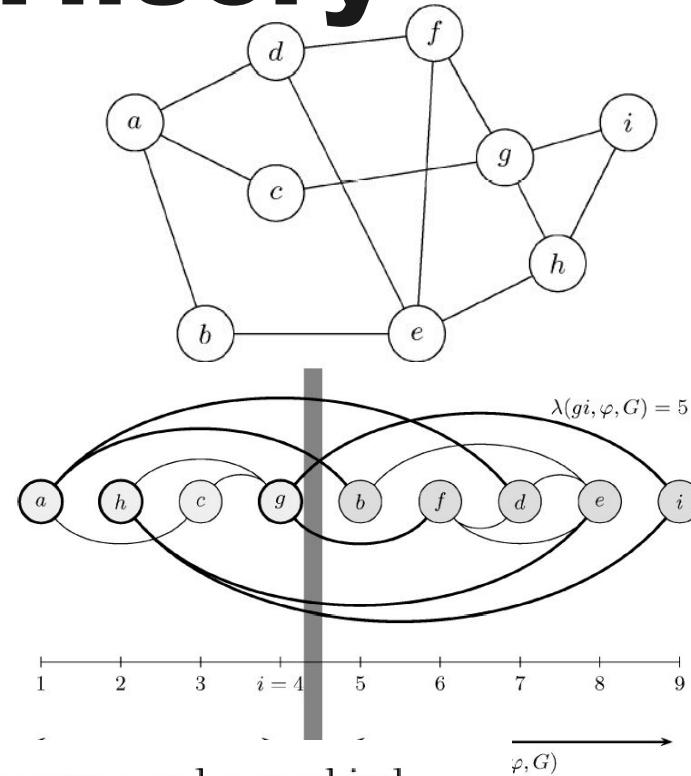
# Idiom: circular layouts / arc diagrams (node-link)

- restricted node-link layouts: lay out nodes around circle or along line
- data
  - original: network
  - derived: node ordering attribute (global computation)
- considerations: node ordering crucial to avoid excessive clutter from edge crossings
  - examples: before & after barycentric ordering



# Linear Layout Theory

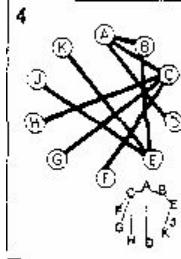
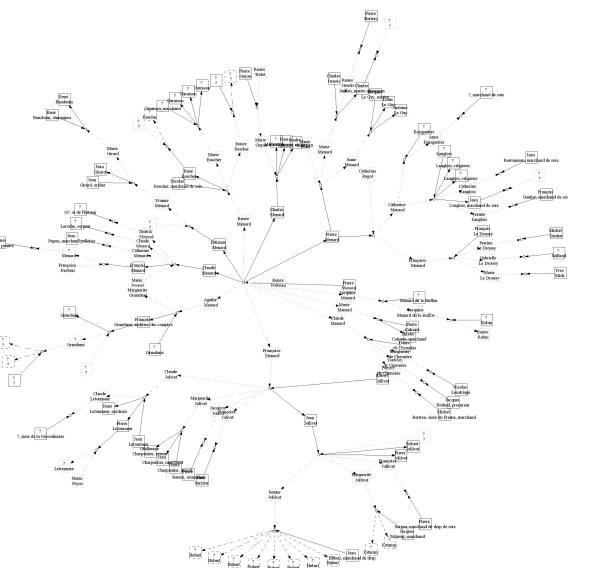
- Order vertices so that a structure appears
  - Layout  $\varphi$
- Some well-known objective functions (Diaz et al. 02):
  - Bandwidth
  - MinLA
  - Cut Width
  - Min Cut
  - Sum Cut



**Fig. 1.** A graph  $G$  together with some layout measures and a graphical representation of the layout  $\varphi = \{(a, 1), (b, 5), (c, 3), (d, 7), (e, 8), (f, 6), (g, 4), (h, 2)\}$ .

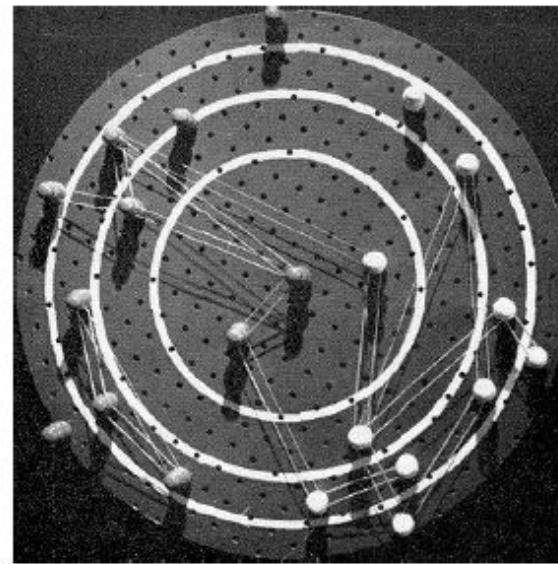
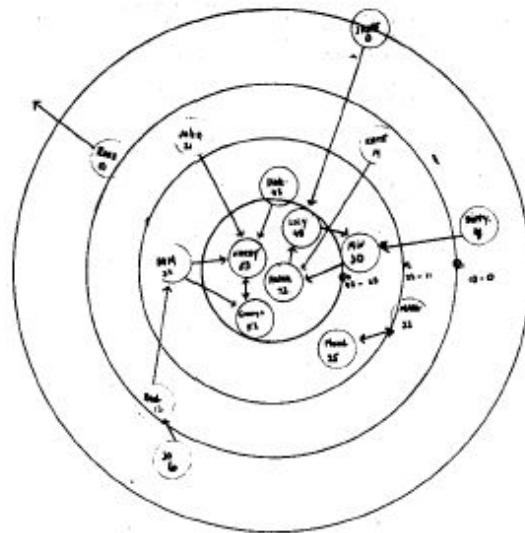
# Circular Layout

- Ego-centric Social Networks
  - Can be ordered too



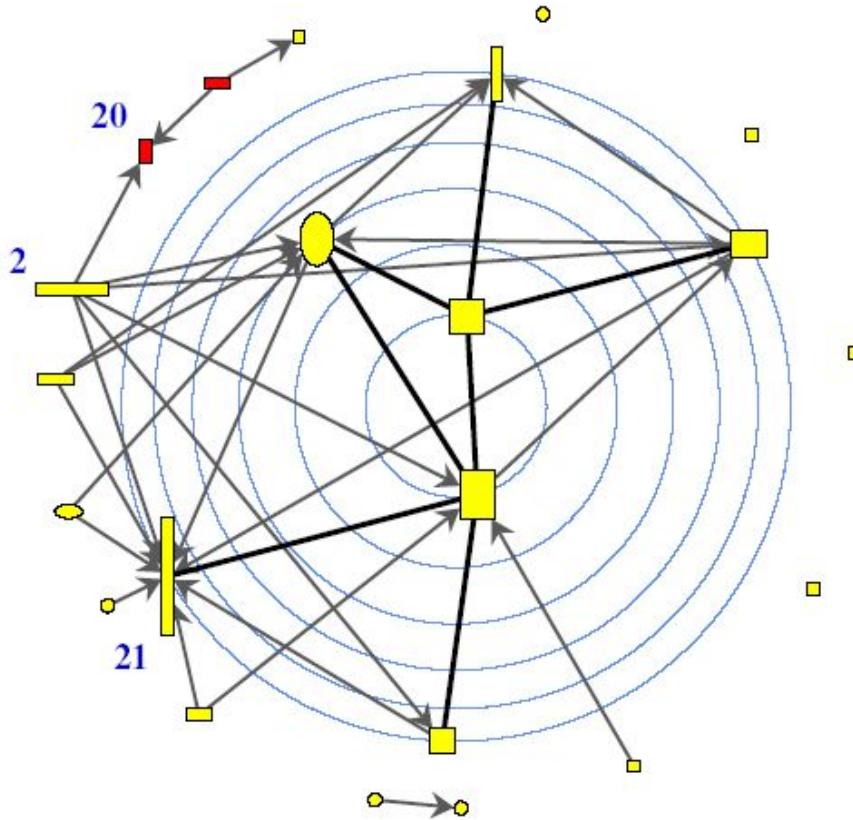
# centrality visualization

Northway (1940)



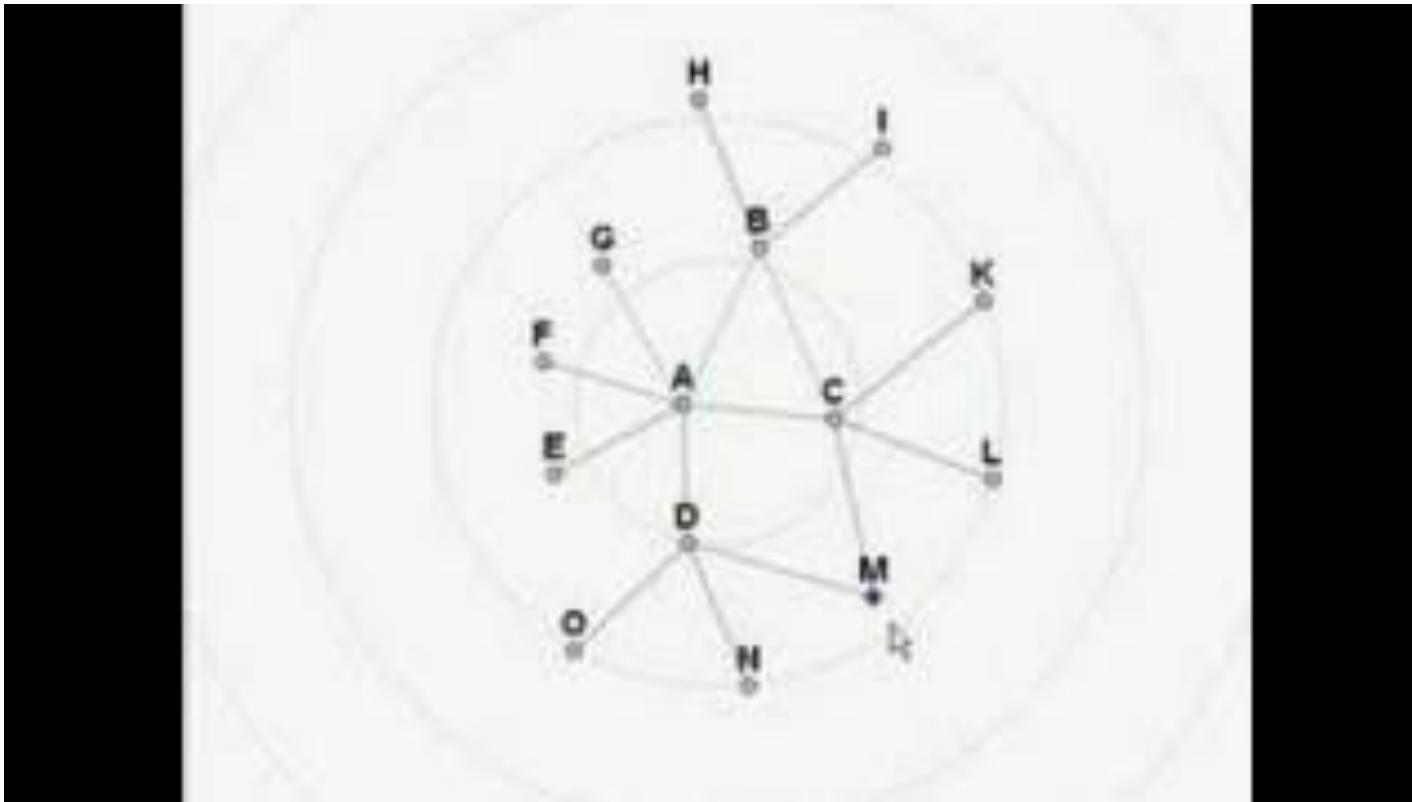
# centrality visualization

network of organizations involved in policy making



# Animation of a circular network

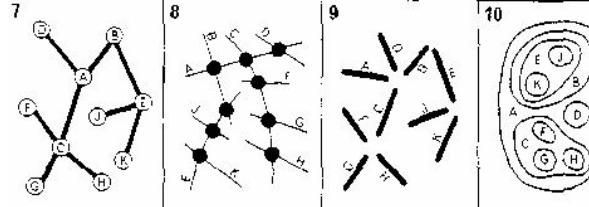
[Animated Exploration of Graphs with Radial Layout, InfoVis'01]



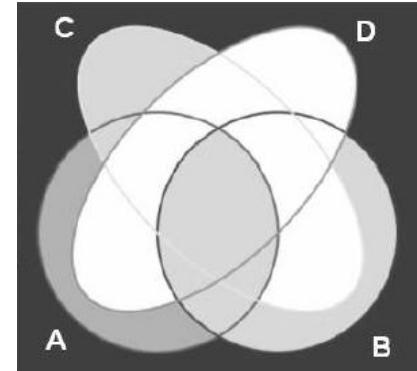
[https://youtu.be/OPX5iGro\\_IA](https://youtu.be/OPX5iGro_IA)

# Free Layout

- Order is not meaningful, only proximity is
- Force-directed methods
- Treemaps and Venn Diagrams

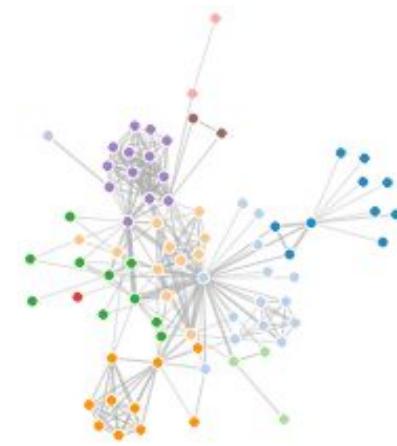


```
for  $t \leftarrow 1$  to  $ITERATIONS$  do
  for  $v \in V$  do
     $D \leftarrow \sum_{u: \{u,v\} \notin E} f_{rep}(p_u, p_v) + \sum_{u: \{u,v\} \in E} f_{spring}(p_u, p_v);$ 
     $p_v \leftarrow p_v + \delta \cdot D$ 
```



# Idiom: **force-directed placement**

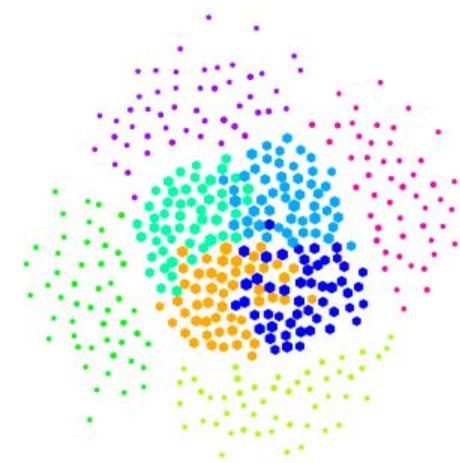
- visual encoding
  - link connection marks, node point marks
- considerations
  - spatial position: no meaning directly encoded
    - left free to minimize crossings
  - proximity semantics?
    - sometimes meaningful
    - sometimes arbitrary, artifact of layout algorithm
    - tension with length
      - long edges more visually salient than short
- tasks
  - explore topology; locate paths, clusters
- scalability
  - node/edge density  $E < 4N$



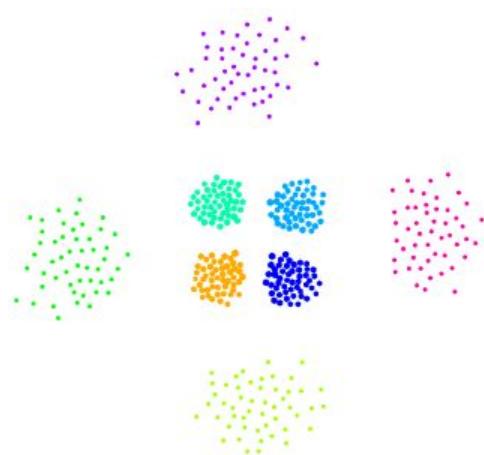
# Force-directed Layout

- Spring Model:  $f_{\text{rep}}$  linear,  $f_{\text{att}}$  linear
  - Good space filling when the network is sparse and homogeneous
- Kamada&Kawai :  $f_{\text{rep}}$  linear,  $f_{\text{att}}$  function of the Graph-theoretical distance
  - Shows the shortest paths as quasi straight lines
- Fruchterman&Reingold :  $f_{\text{att}}$  quadratic
  - Fills the space better for varying degrees
- Davidson&Harel :  $f_{\text{rep}}$  linear,  $f_{\text{att}}$  linear\*degree
  - Better space filling for heterogeneous graphs
- Hall/Spectral
  - Very fast but works best with grids
- LinLog :  $f_{\text{rep}}$  linear,  $f_{\text{att}}$  linear+log
  - Show clusters
- New algorithms all the time
- Now, merging with multidimensional projections

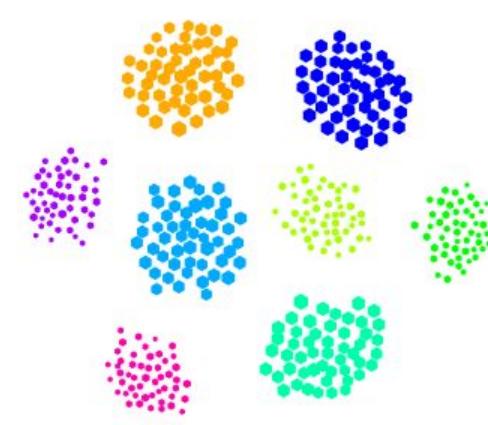
# Examples



Fruchterman-Reingold model

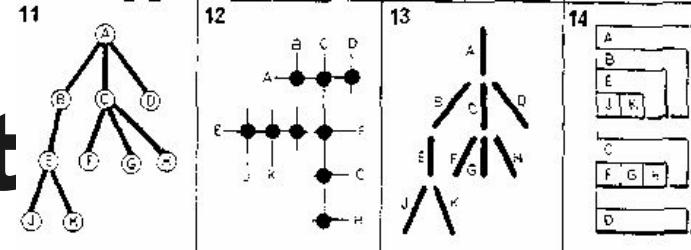


Node-repulsion LinLog

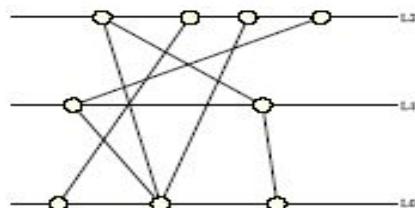


Edge-repulsion LinLog model

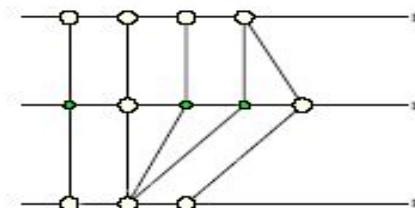
# Ordered layout



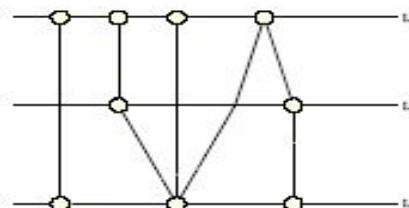
- One dimension is orderable (e.g. genealogical trees)
  - The other can be partially ordered
- Sugiyama algorithm
- Improved by C. North [A Technique for Drawing Directed Graphs.]



vertical ordering  
layer assignment



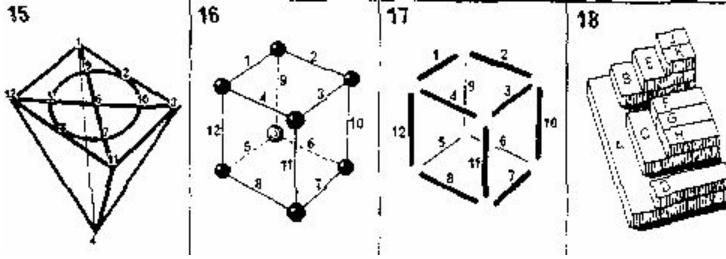
horizontal ordering  
crossing reduction



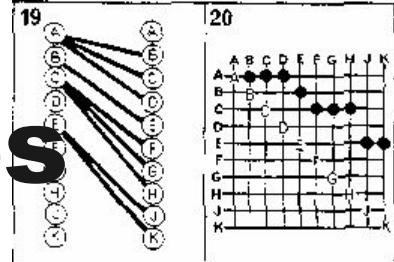
coordinate assignment  
spacing and straightness

# 3D Networks

- 3D methods usually extend to 3D naturally
- Navigation is then required



# Diagrams and Matrices



Directed network

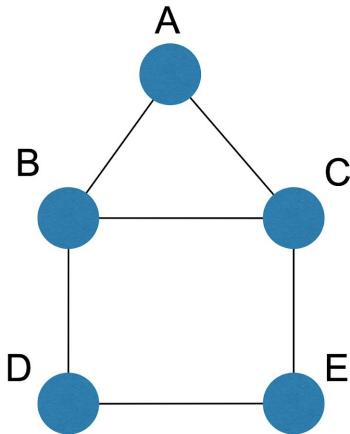


Adjacency matrix

	A	B	C	D	<i>Destination/target</i>
A	0	1	0	0	
B	0	0	1	0	
C	0	0	0	1	
D	0	0	0	0	
<i>Source</i>					

# Adjacency matrix representations

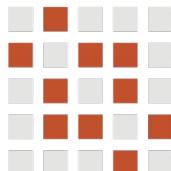
- derive adjacency matrix from network



	A	B	C	D	E
A					
B					
C					
D					
E					

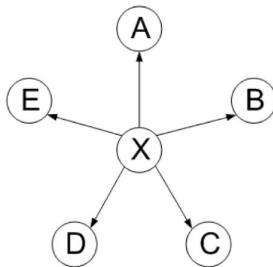
→ **Adjacency Matrix**  
Derived Table

✓ NETWORKS ✓ TREES

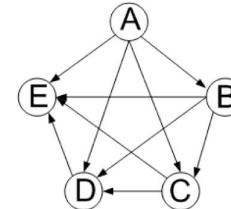


# Adjacency matrix examples

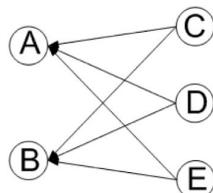
	E	D	C	B	A
...	X	Y	Z	...	



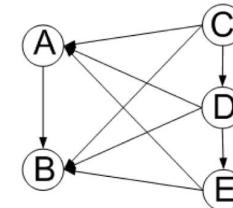
	E	D	C	B	A
A					
B					
C					
D					
E					



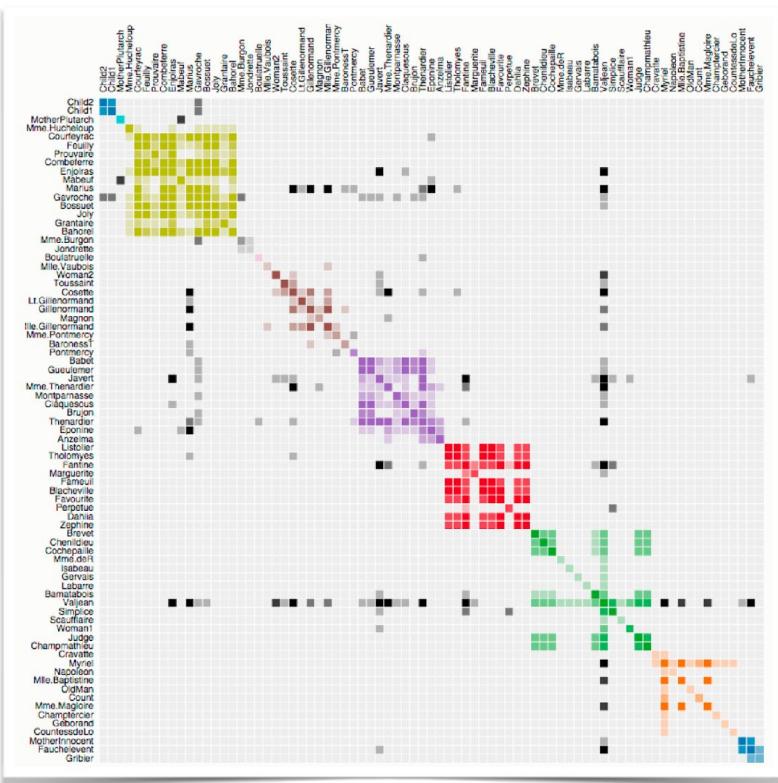
	E	D	C	B	A
A					
B					
C					
D					
E					



	E	D	C	B	A
A					
B					
C					
D					
E					

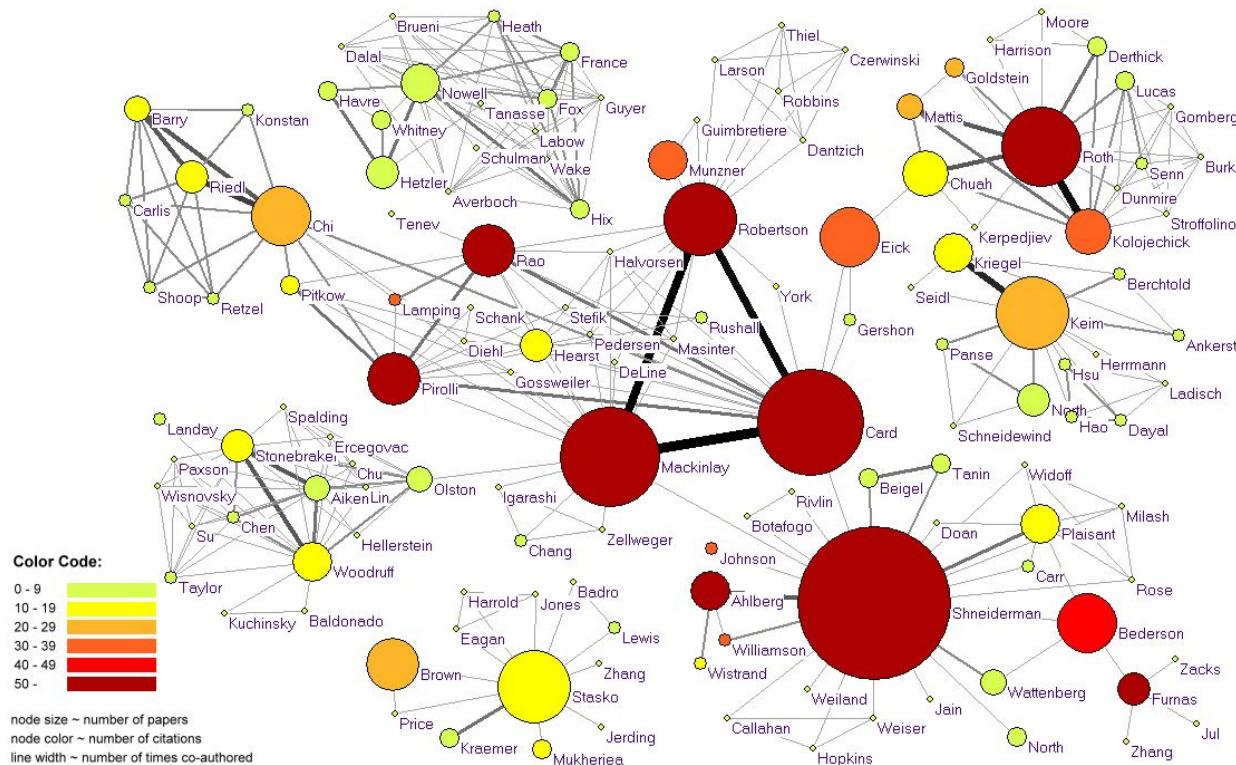


# Node order is crucial: Reordering

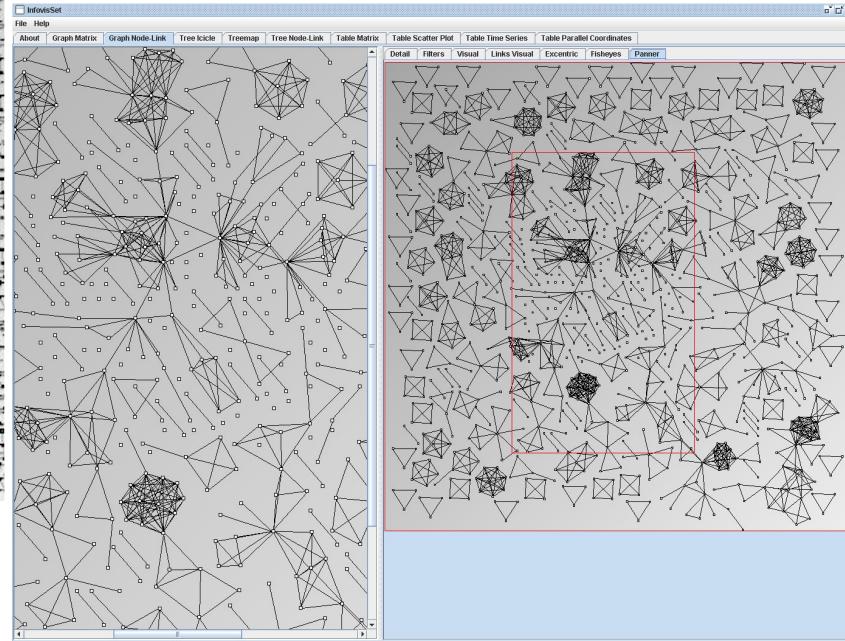
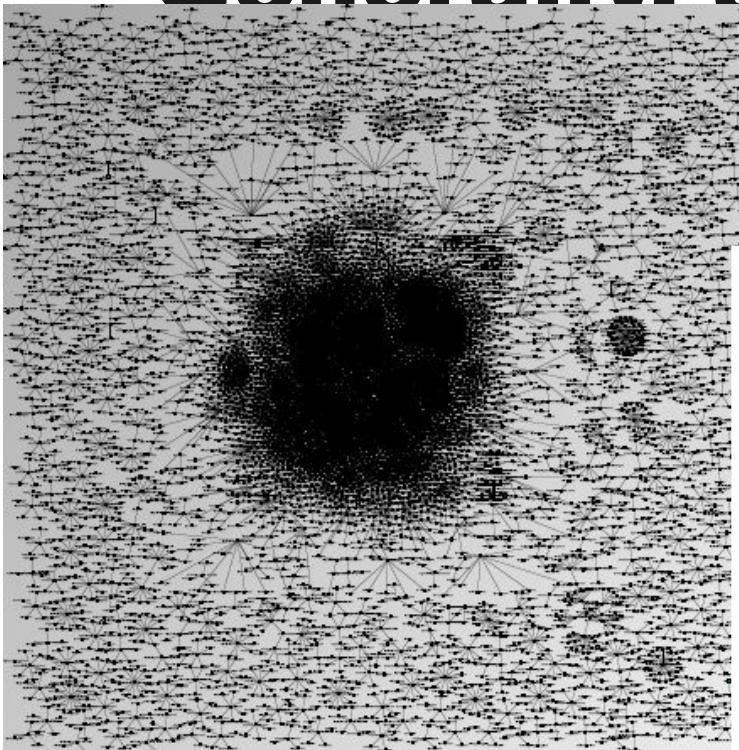


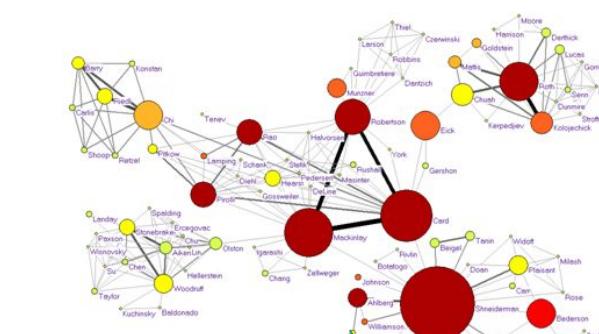
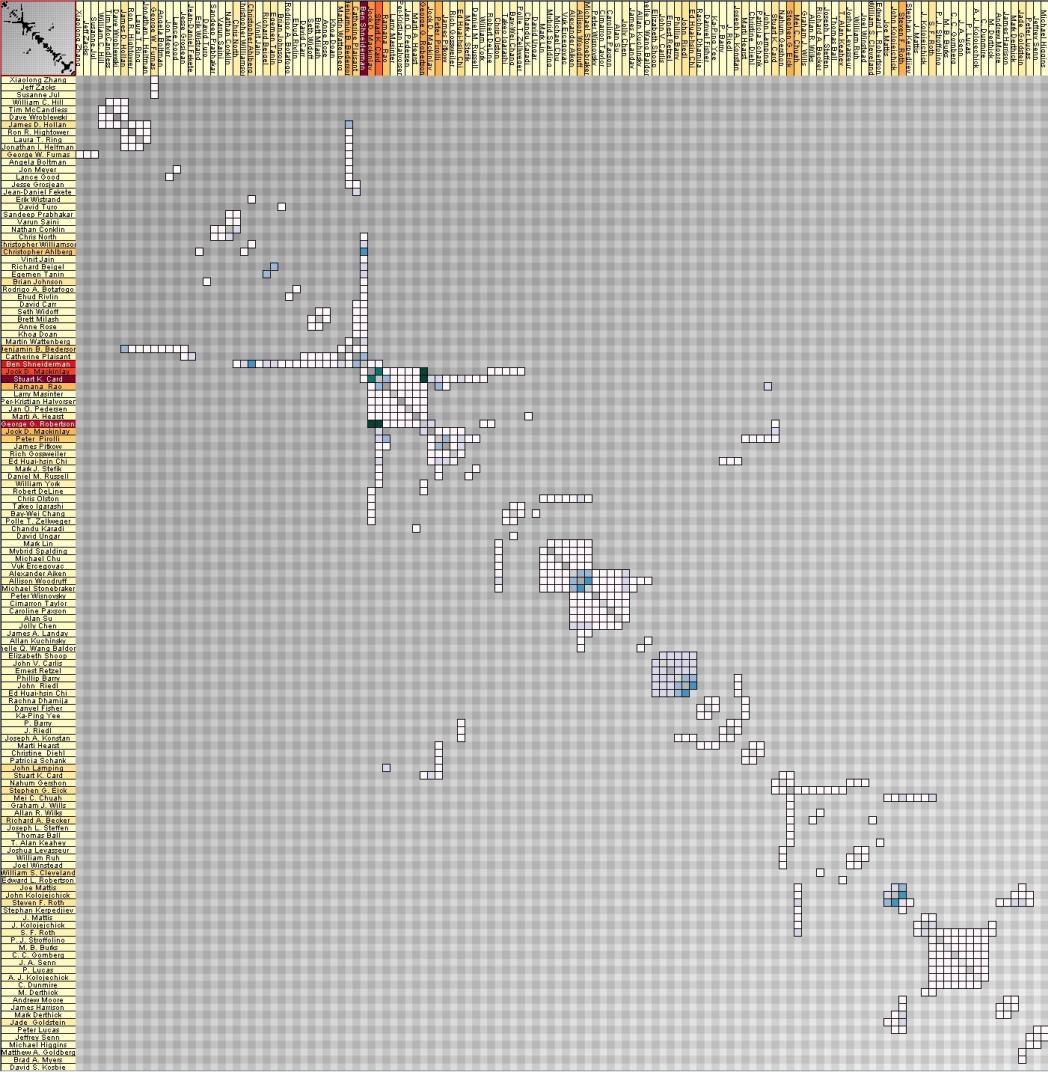
<https://bostocks.org/mike/miserables/>

# InfoVis Co-authoring (K. Börner et



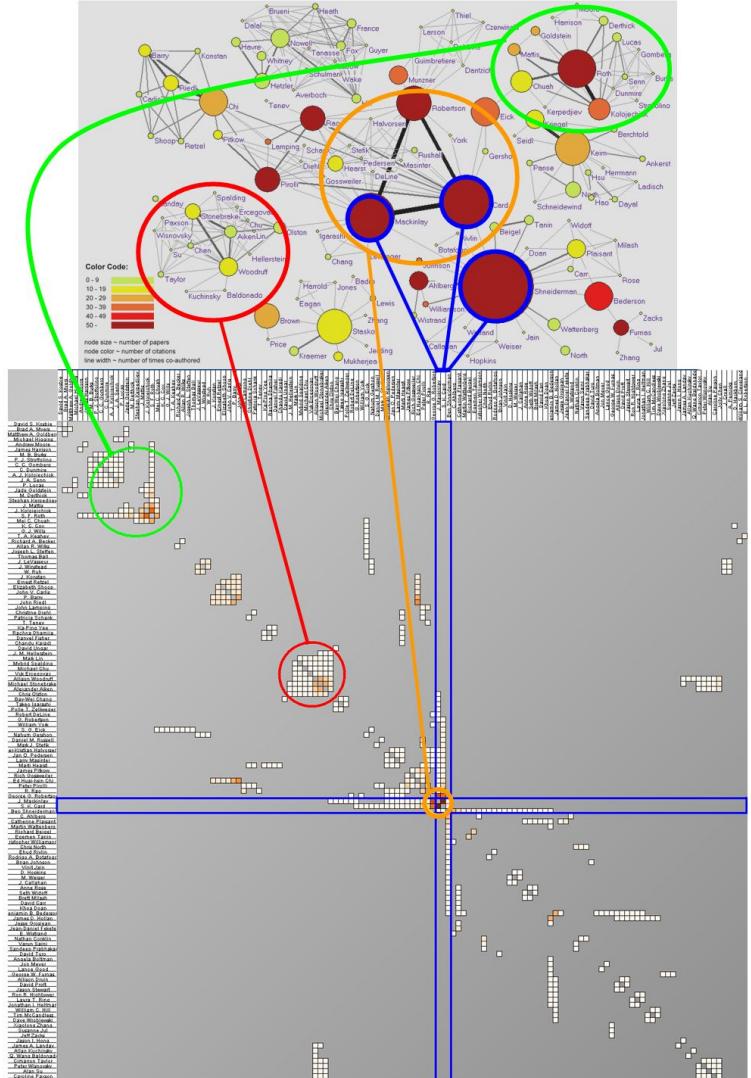
# Generally, after loading...



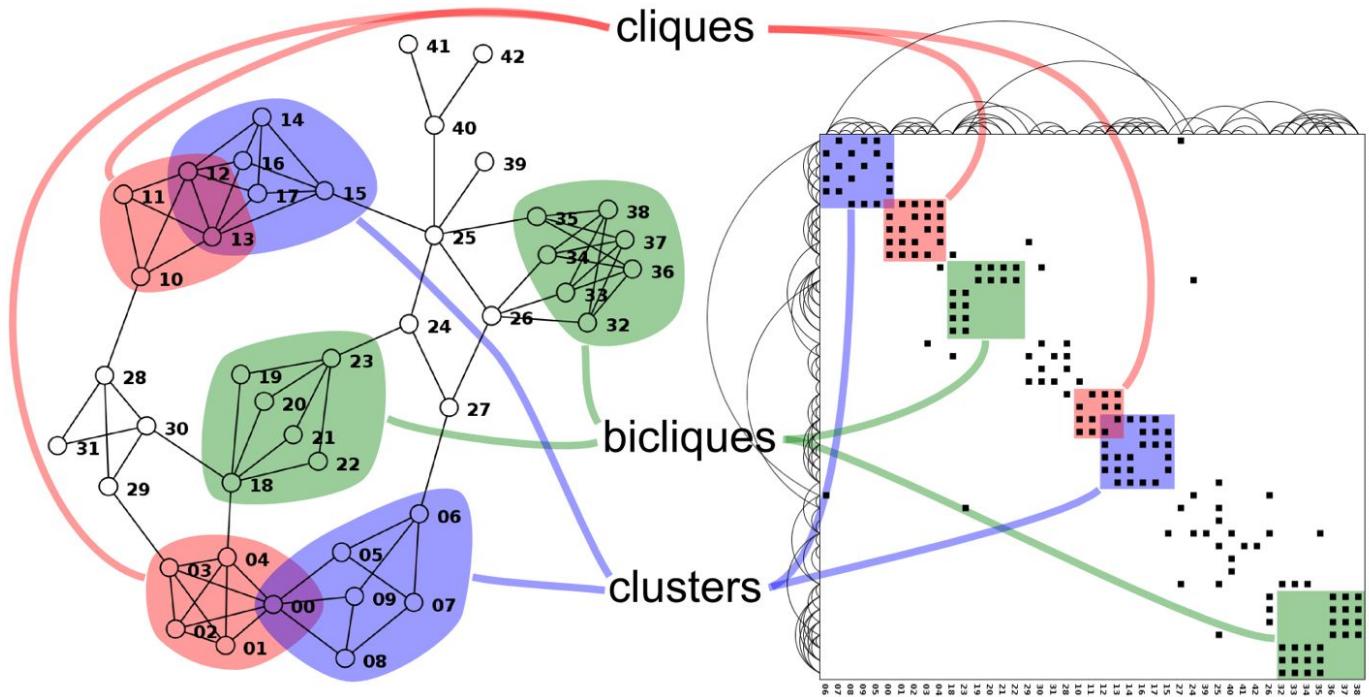


Communicate

Explore

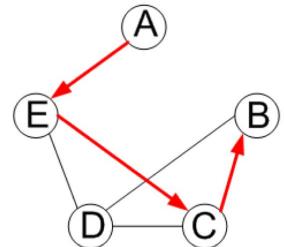


# Structures visible in both



# Adjacency matrix

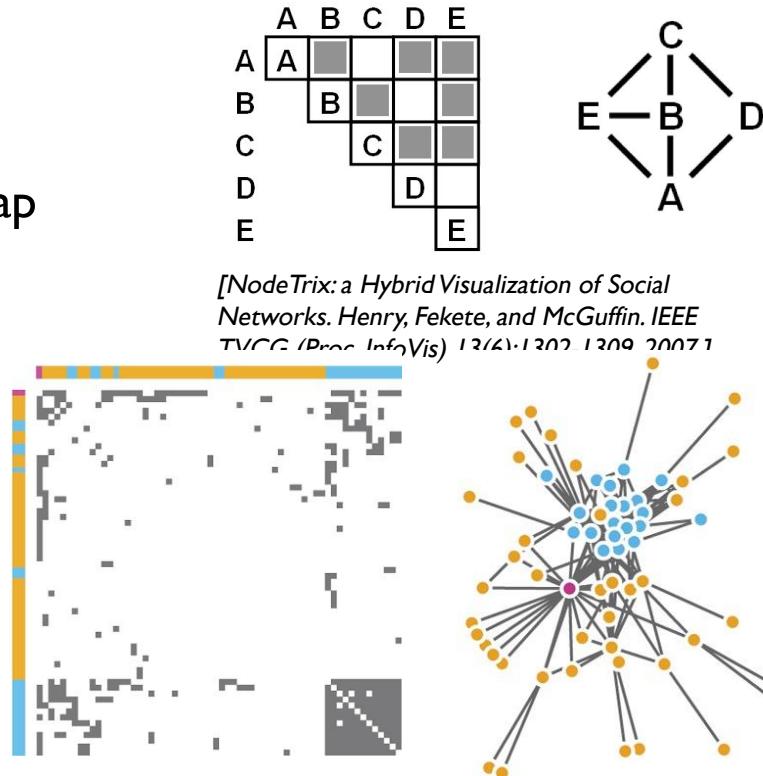
	TO							
A		B	C	D	E	F	G	H
A								
B								
C								
F								
D								
R								
O								
M								
E								
F								
G								
H								



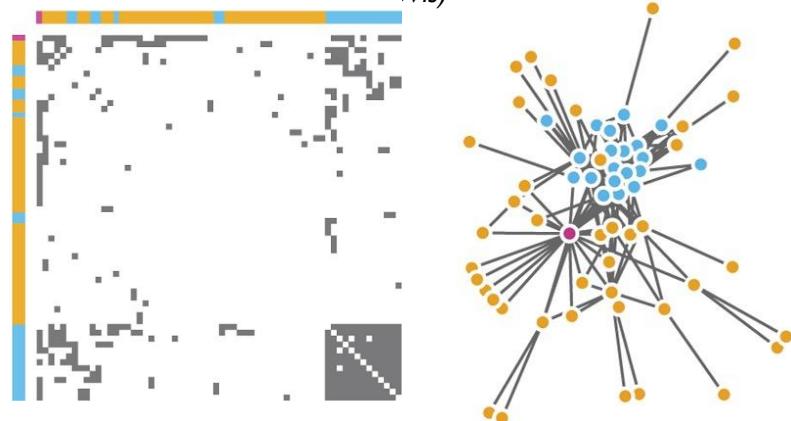
	A	B	C	D	E
E					
D					
C					
B					
A					

# Idiom: adjacency matrix view

- data: network
  - transform into same data/encoding as heatmap
- derived data: table from network
  - 1 quant attrib
    - weighted edge between nodes
  - 2 categ attribs: node list x 2
- visual encoding
  - cell shows presence/absence of edge
- scalability
  - 1K nodes, 1M edges



[*NodeTrix: a Hybrid Visualization of Social Networks*. Henry, Fekete, and McGuffin. IEEE TVCG (Proc. InfoVis) 13(4):1202-1209, 2007.]

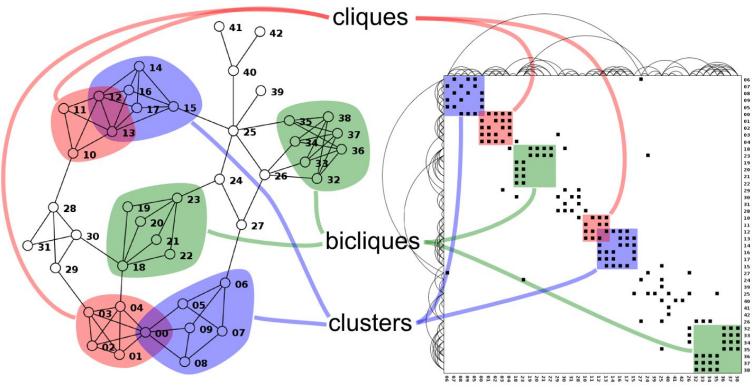


[*Points of view: Networks*. Gehlenborg and Wong. Nature Methods 9:1115.]

# Node-link vs. matrix comparison

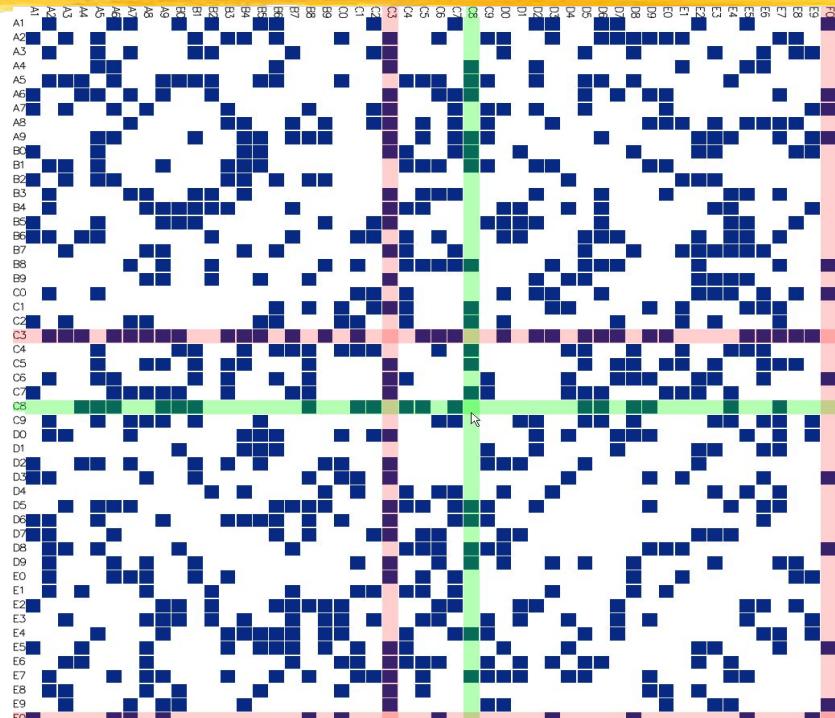
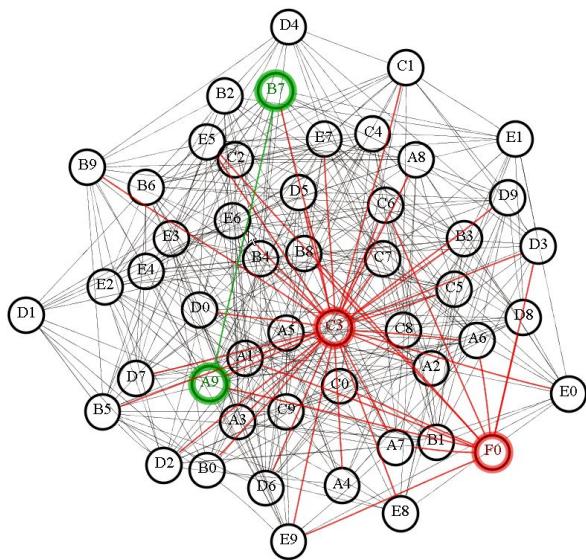
- node-link diagram strengths
  - topology understanding, path tracing
  - intuitive, flexible, no training needed
- adjacency matrix strengths
  - focus on edges rather than nodes
  - layout straightforward (reordering needed)
  - predictability, scalability
  - some topology tasks trainable
- empirical study
  - node-link best for small networks
  - matrix best for large networks
    - if tasks don't involve path tracing!

[On the readability of graphs using node-link and matrix-based representations: a controlled experiment and statistical analysis. Ghoniem, Fekete, and Castagliola. *Information Visualization* 4:2 (2005), 114–135.]



<http://www.michaelmcguffin.com/courses/vis/patternsInAdjacencyMatrix.png>

# Readability Experiment



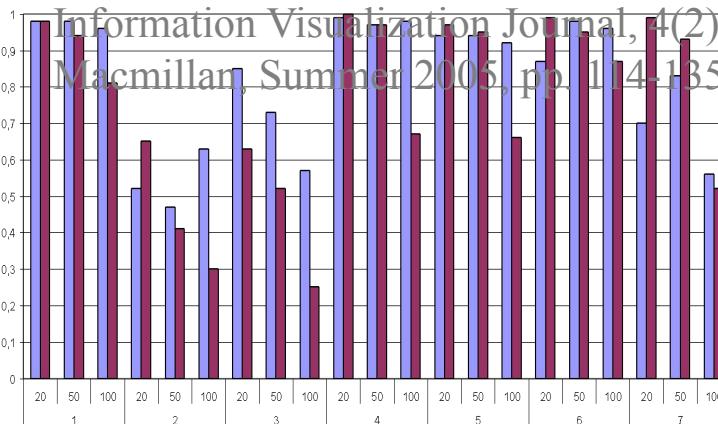
# Controlled Experiment: Node Link Diagrams vs. Adjacency Matrices

- **The Tasks:**
- Tasks related to the overview
  - Number of vertices
  - Number of arcs
- Tasks related to graph elements
  - Finding an element (a vertex, a link)
  - Finding the most connected vertex (a central actor, a pivot, a hub)
  - Finding a common neighbor
  - Finding a path
- Random graphs (3 sizes et 3 densities)
- 2 representations: Node-Link + Matrix
- **Results:**
- Node-link diagrams are preferable for small sparse graphs (20 vertices)

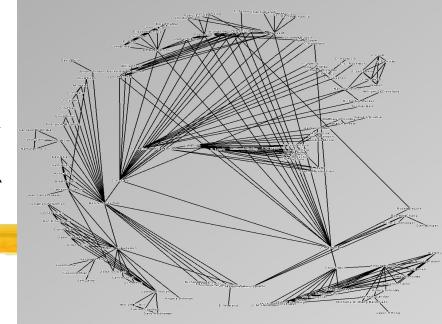
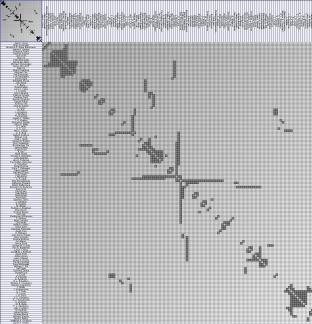
Matrices are more readable wrt dense graphs and medium/large graphs (> 20 vertices) wrt the

## References:

Mohammad Ghoniem, Jean-Daniel Fekete and Philippe Castagliola *Readability of Graphs Using Node-Link and Matrix-Based Representations: Controlled Experiment and Statistical Analysis*, Information Visualization Journal, 4(2), Palgrave Macmillan, Summer 2005, pp. 114-135.



Completion time for the 7 tasks, 3 densities and 2 representations (Node-Link in blue, Matrix in red)



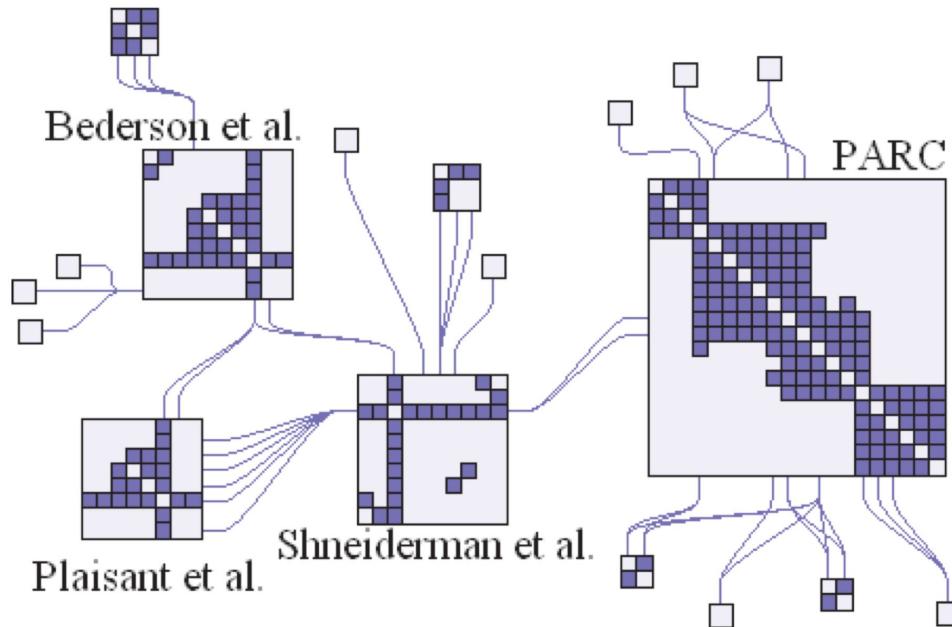
# Matrix vs. NodeLink

- Usable without reordering
- No node overlapping
- No edge crossing
  - Readable for dense graphs
- Fast navigation
- Fast manipulation
  - Usable interactively
- More readable for some tasks
- Less familiar
- Use more space
- Weak for path following tasks

- Familiar
  - Compact
  - More readable for path following
  - More effective for small graphs
  - More effective for sparse graphs
- 
- Useless without layout
  - Node overlapping
  - Edge crossing
    - Not readable for dense graphs
  - Manipulation requires layout

# Idiom: NodeTrix

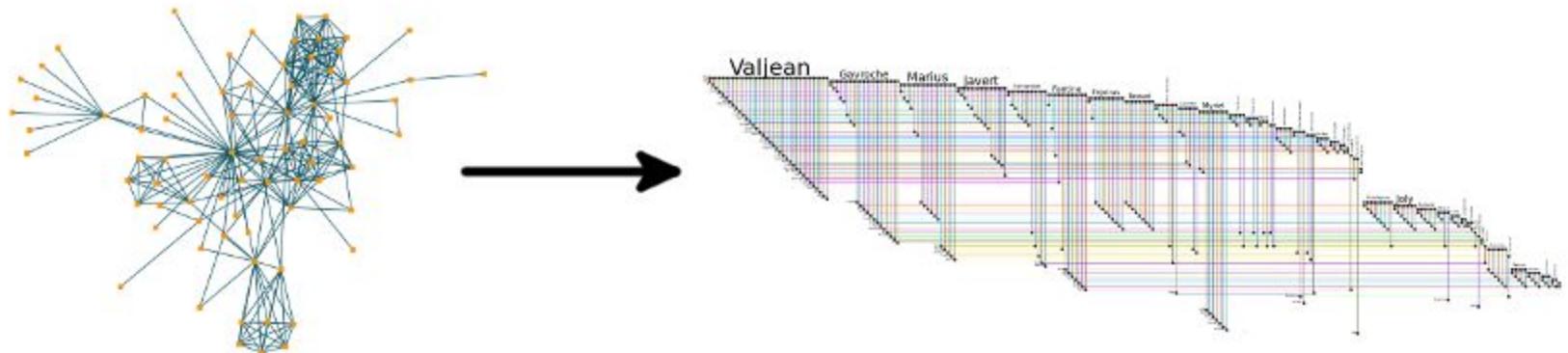
- hybrid nodelink/matrix
- capture strengths of both



[*NodeTrix: a Hybrid Visualization of Social Networks. Henry, Fekete, and McGuffin. IEEE TVCG (Proc. InfoVis) 13(6):1302-1309, 2007.*]

---

## More Graph Drawing: Biofabric



<https://biofabric.systemsbiology.net/gallery/pages/SuperQuickBioFabric.html>

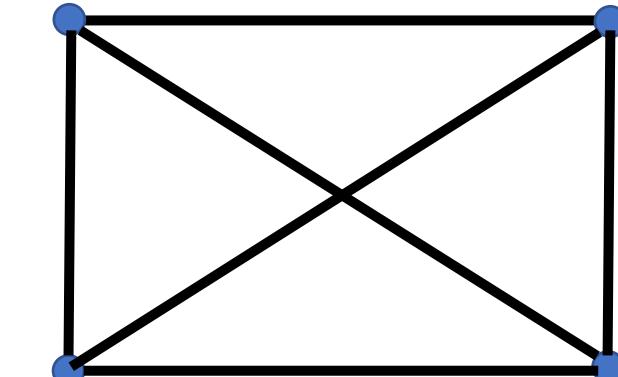
# Hypergraph?

= a generalization of a graph  
where EDGES can connect  
one, two, or more VERTICES

# **Graph (e.g. who knows who)**

Catherine

Jean-Daniel



Paolo

Paola

Catherine — Jean-Daniel

Catherine — Paolo

Catherine — Paola

Paolo — Paola

Paolo — Jean-Daniel

Paola — Jean-Daniel

Node link diagram

# Hypergraph (e.g. teams)

Catherine Jean-Daniel

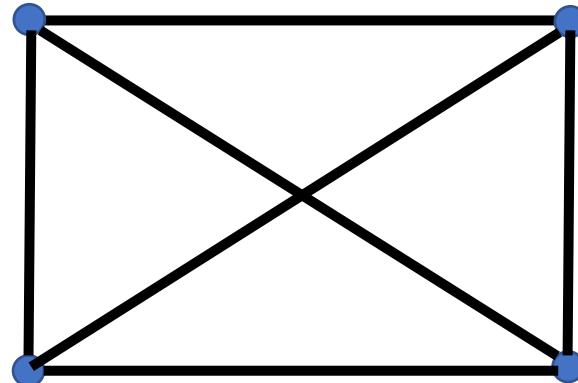
Catherine Paolo

Paolo Paola Jean-Daniel

Paolo Paola Jean-Daniel Catherine

Catherine

Jean-Daniel



Paolo

Paola

Catherine    Jean-Daniel

Catherine    Paolo

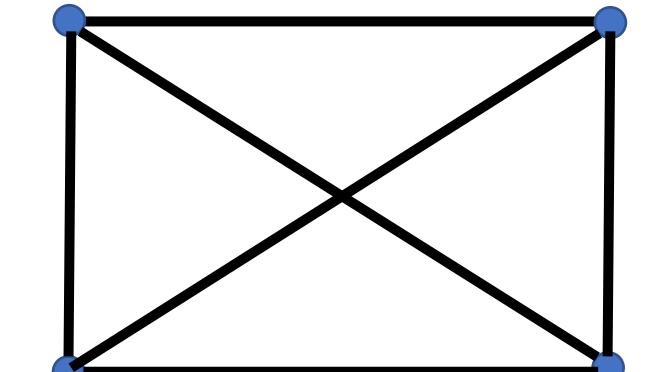
Paolo    Paola    Jean-Daniel

Paolo    Paola    Jean-Daniel    Catherine

Node link diagram

Catherine

Jean-Daniel



Paolo

Paola

Catherine    Jean-Daniel

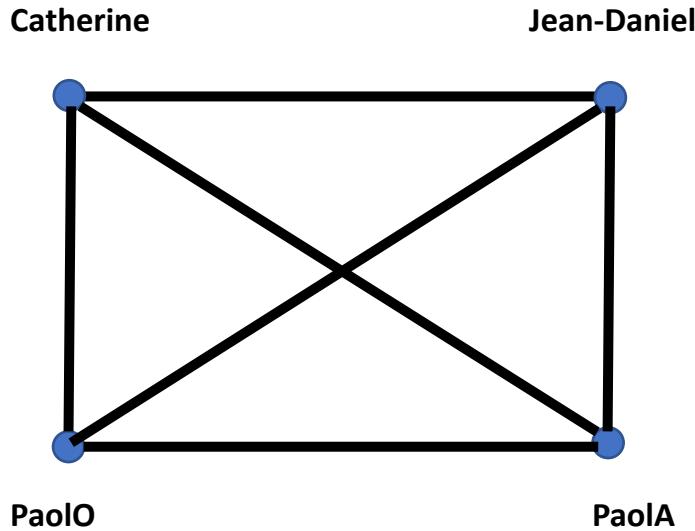
Catherine    Paolo

Paolo    Paola    Jean-Daniel

Paolo    Paola    Jean-Daniel    Catherine

4 teams / 4 hyperedges

Looks just the same  
as...

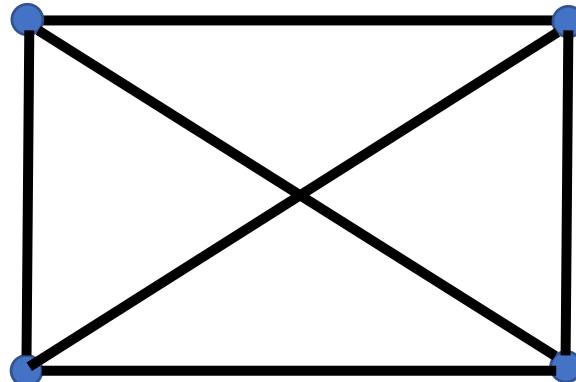


Catherine	—	Jean-Daniel
Catherine	—	Paolo
Catherine	—	Paola
Paolo	—	Paola
Paolo	—	Jean-Daniel
Paola	—	Jean-Daniel

Looks just the same as... **the “who knows who” graph**

Catherine

Jean-Daniel



Paolo

Paola

Catherine    Jean-Daniel

Catherine    Paolo

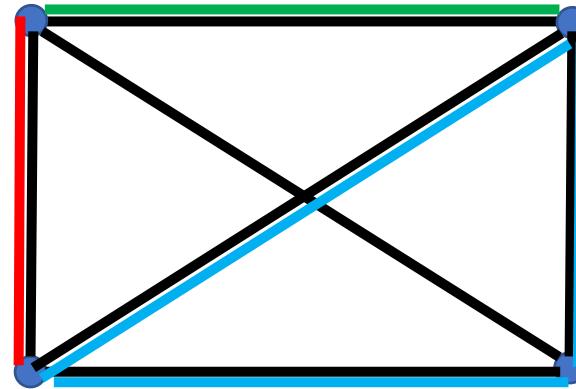
Paolo    Paola    Jean-Daniel

Paolo    Paola    Jean-Daniel    Catherine

**?? How to reveal the hyperedges ??**

Catherine

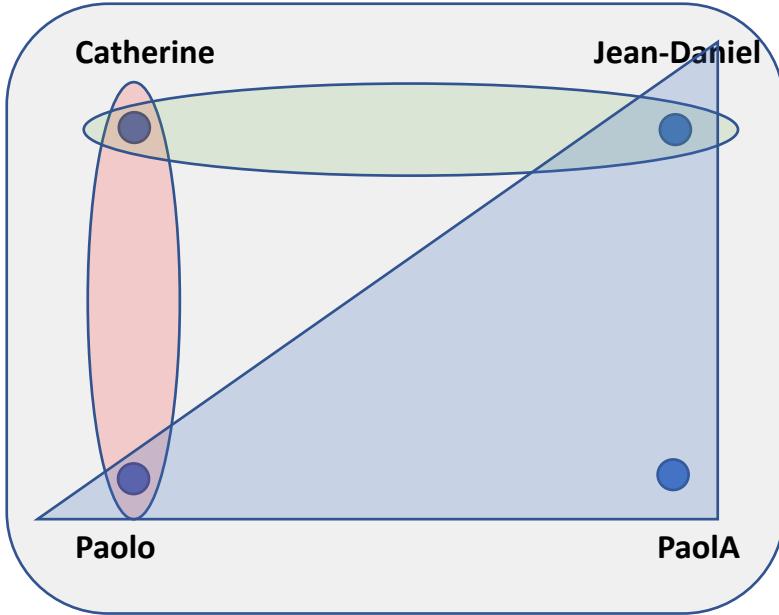
Jean-Daniel



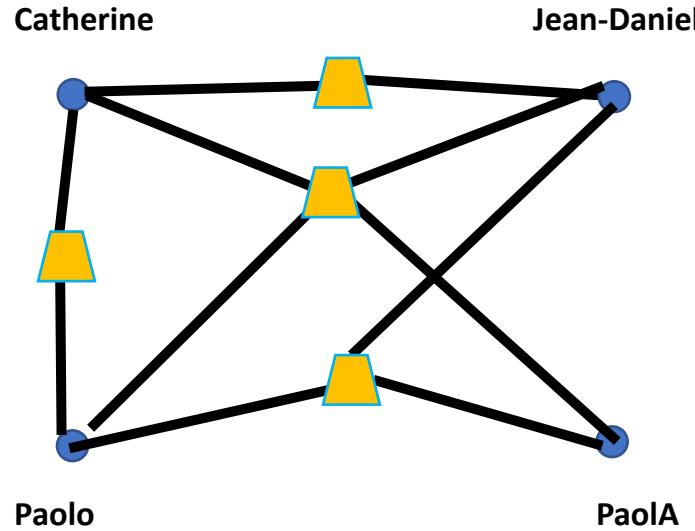
Paolo

Paola

Multiple links – different link attributes

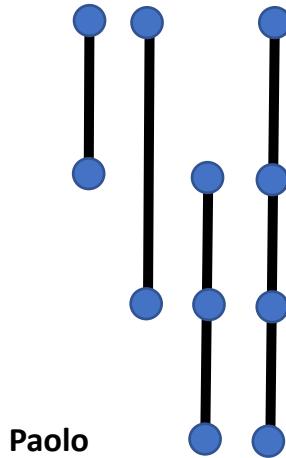


Teams = Sets



Bi-partite network (nodes and teams)

Catherine



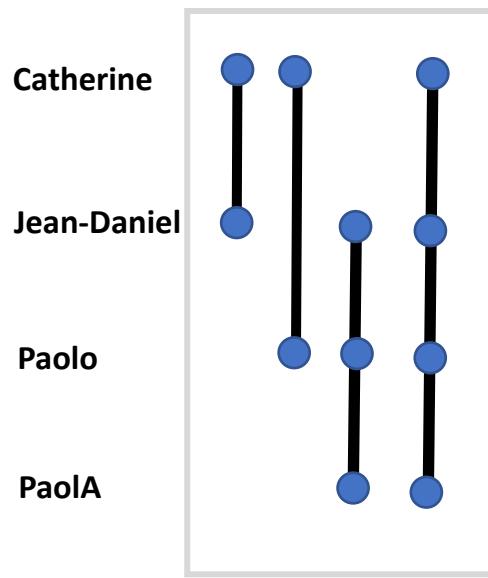
Jean-Daniel

Catherine    Jean-Daniel  
Catherine    Paolo  
Paolo    Paola    Jean-Daniel  
Paolo    Paola    Jean-Daniel    Catherine

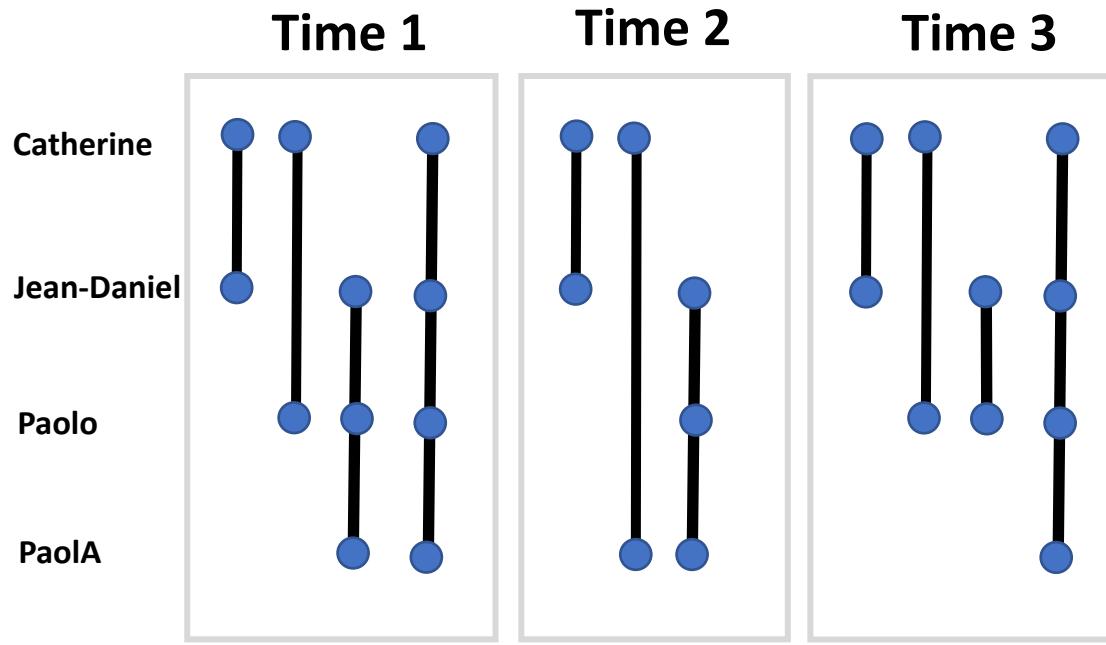
Paola

Our proposed technique: Paola R Valdivia, Paolo Buono, Catherine Plaisant, Nicole Dufournaud, Jean-Daniel Fekete. Analyzing Dynamic Hypergraphs with Parallel Aggregated Ordered Hypergraph Visualization. IEEE Transactions on Visualization and Computer Graphics, 2021, 27 (1), pp.1-13. <10.1109/TVCG.2019.2933196>. [⟨hal-02264960⟩](https://hal-02264960) <https://aviz.fr/paohvis>

**Time 1**

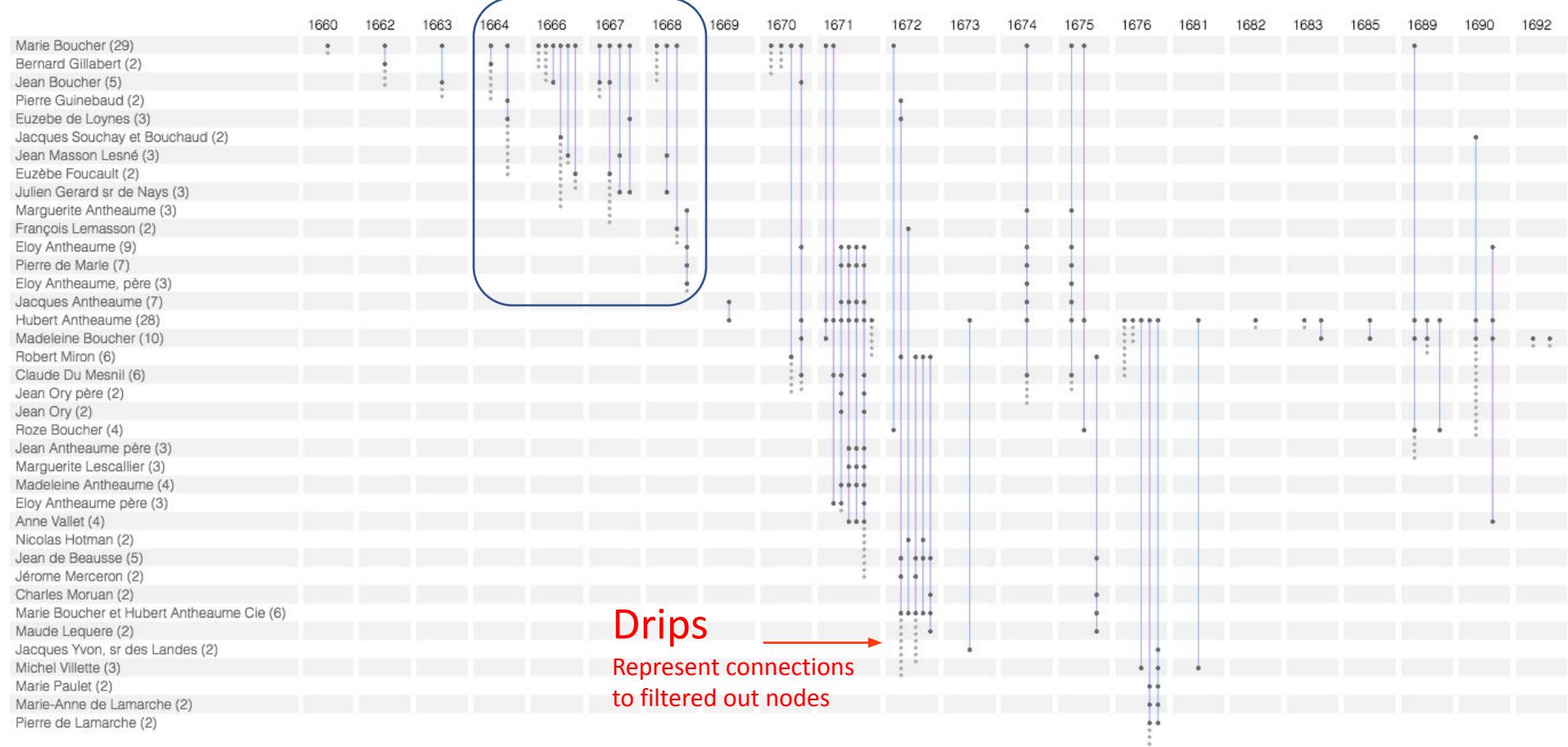


**Hypergraph**



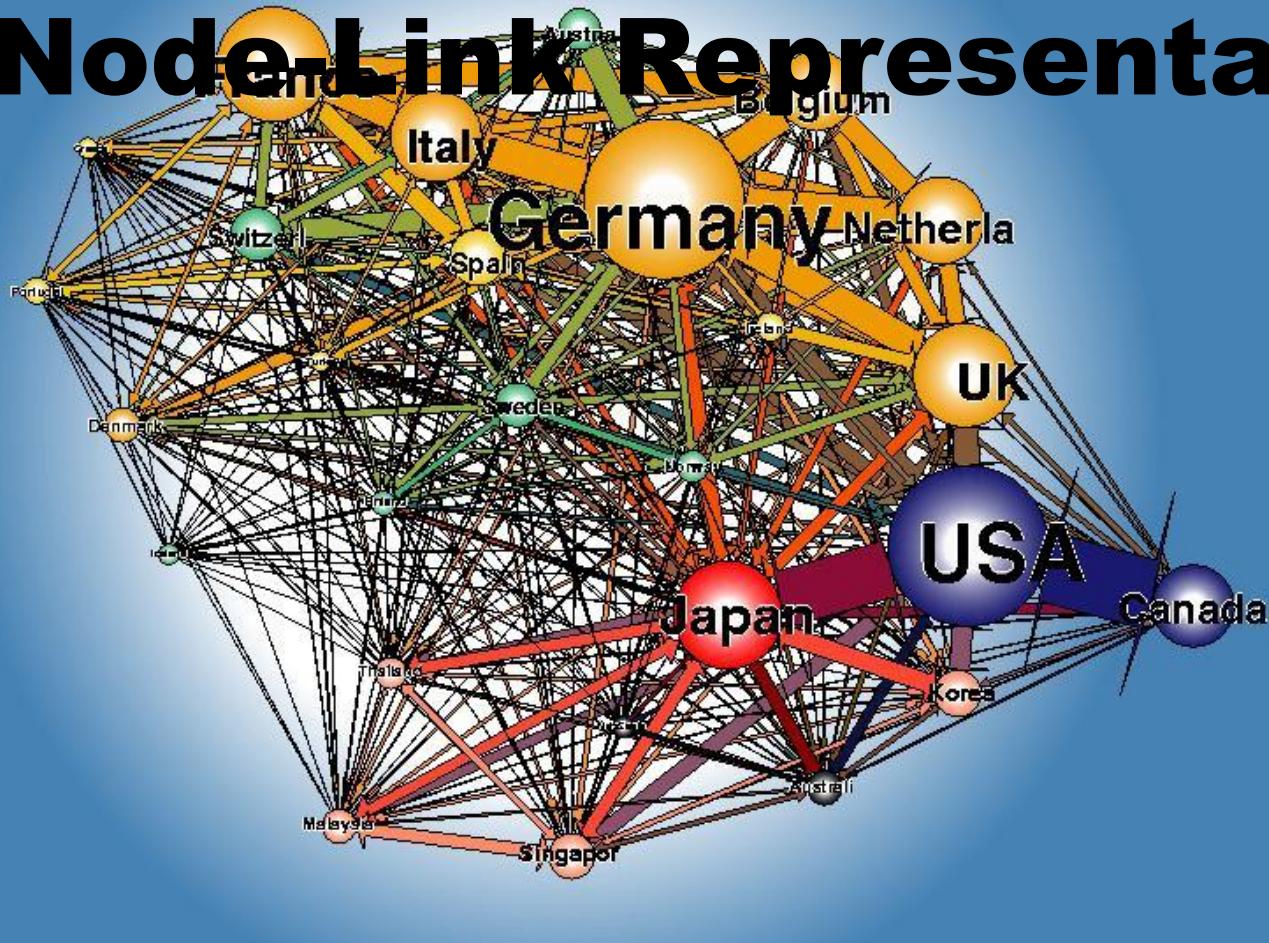
**Dynamic Hypergraph (changes overtime)**

**Parallel Aggregated Ordered Hypergraph**  
**PAOHvis**



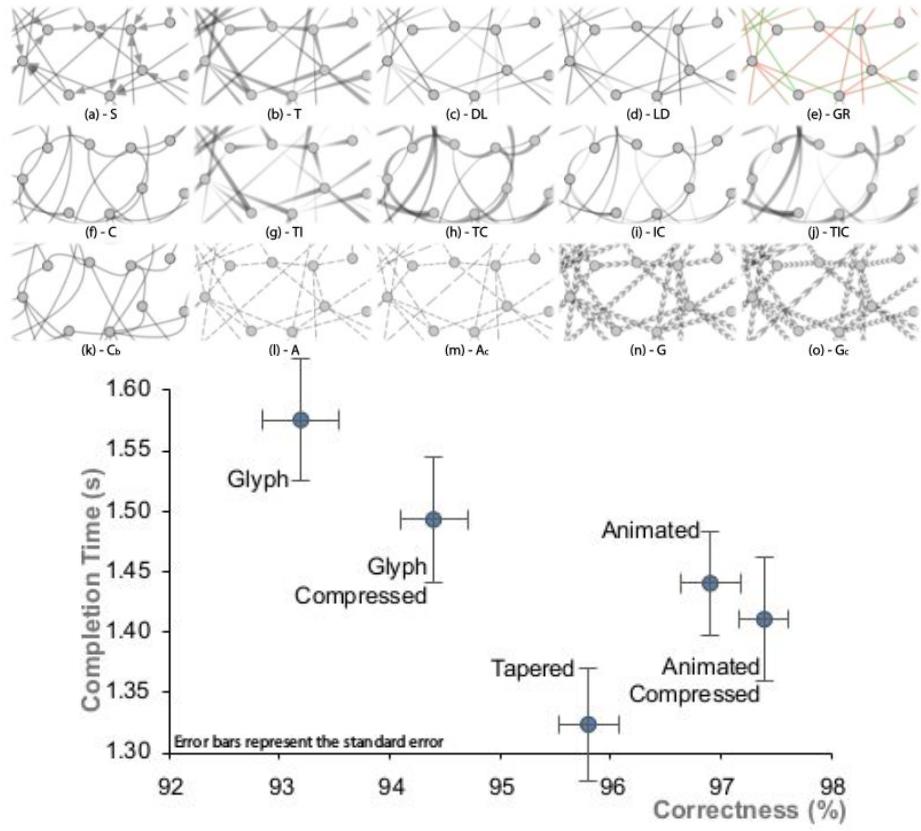
Marie Boucher. 63 documents. 75 people - 48 listed (appear in > 1 document) 28 hinted as "drips"

# Node-Link Representation



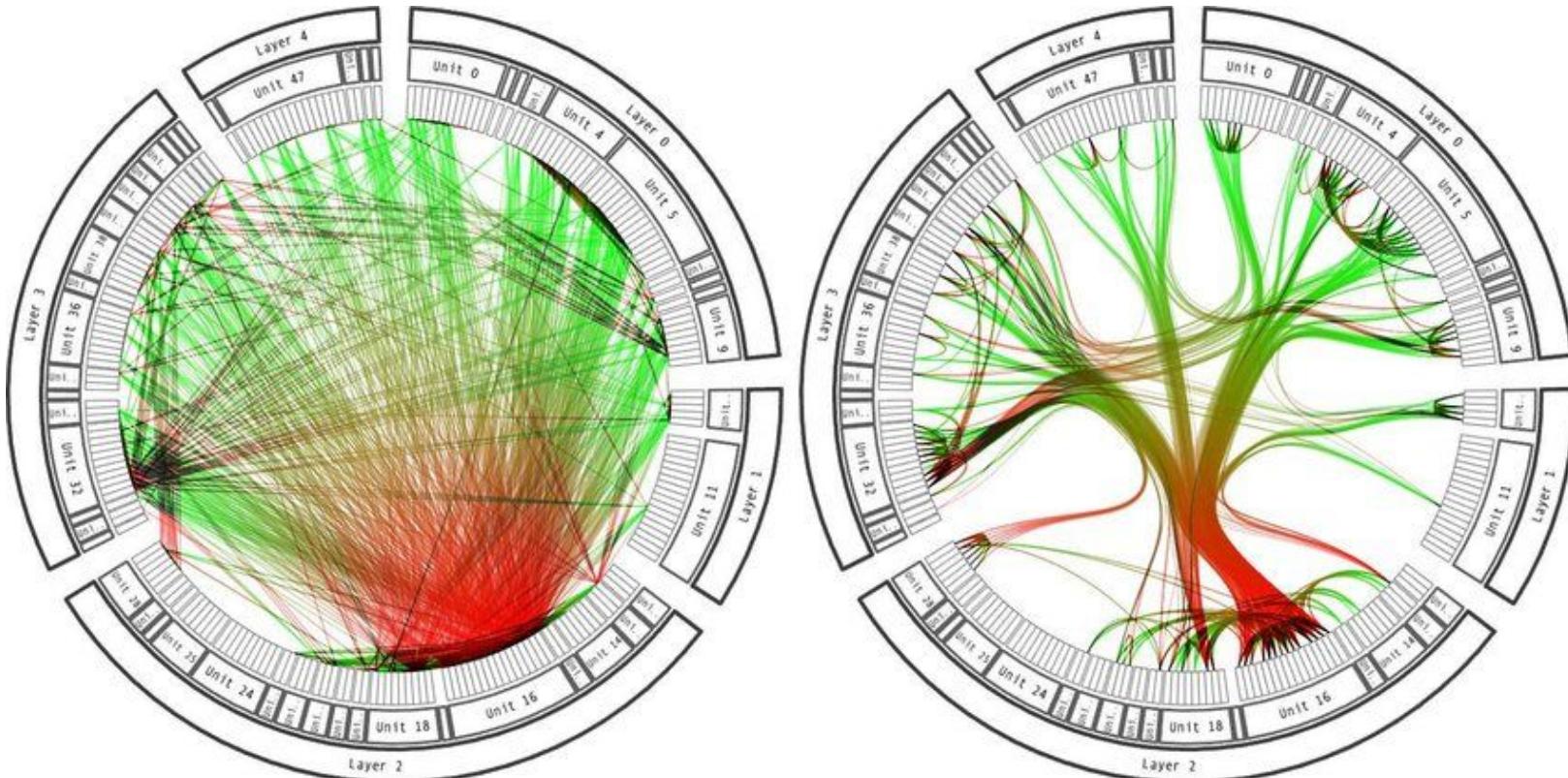
# Lisibilité des liens ?

- 3 codages
- 3 densités
- Plusieur graphes aléatoires
- On pose des questions
- On mesure
  - Le temps de réponse
  - La validité des réponses
- On regarde s'il y a une différence statistique
- Oui !



Danny Holten, Petra Isenberg, Jarke Van Wijk, Jean-Daniel Fekete. An Extended Evaluation of the Readability of Tapered, Animated, and Textured Directed-Edge Representations in Node-Link Graphs. IEEE Press. *Pacific Visualization Symposium (PacificVis)*, 2011 IEEE, Mar 2011, Hong Kong, China. 8p, 2011.

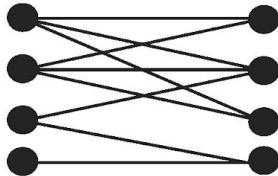
# Edge bundling



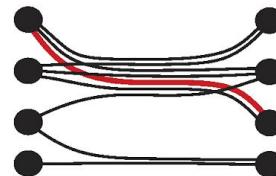
# Edge Bundling ou pas ?

## Edge Bundling Methods

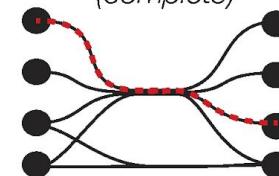
(a) No bundling



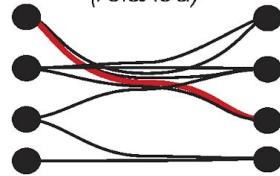
(b) Metro-Style Bundling



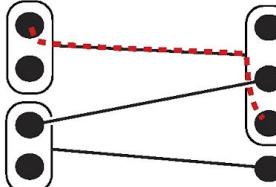
(c) Edge Bundling  
(complete)



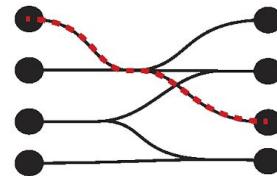
(d) Edge Bundling  
(relaxed)



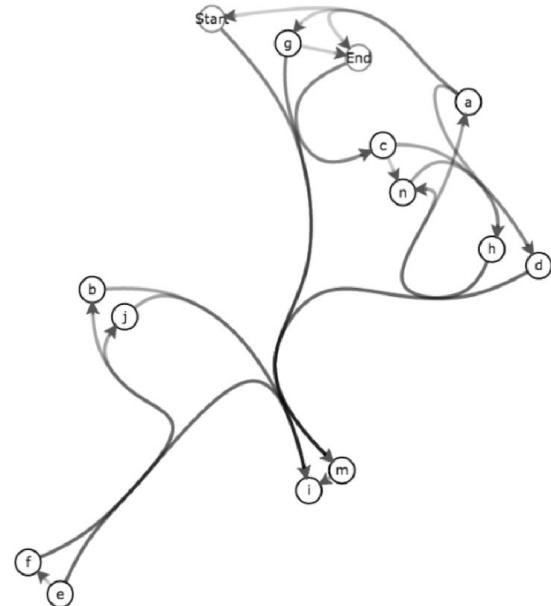
(e) Power Graphs



(f) Confluent Drawing



## Confluent Graph



<https://aviz.fr/~bbach/confluentgraphs/>

# Edge Bundling ou pas ?

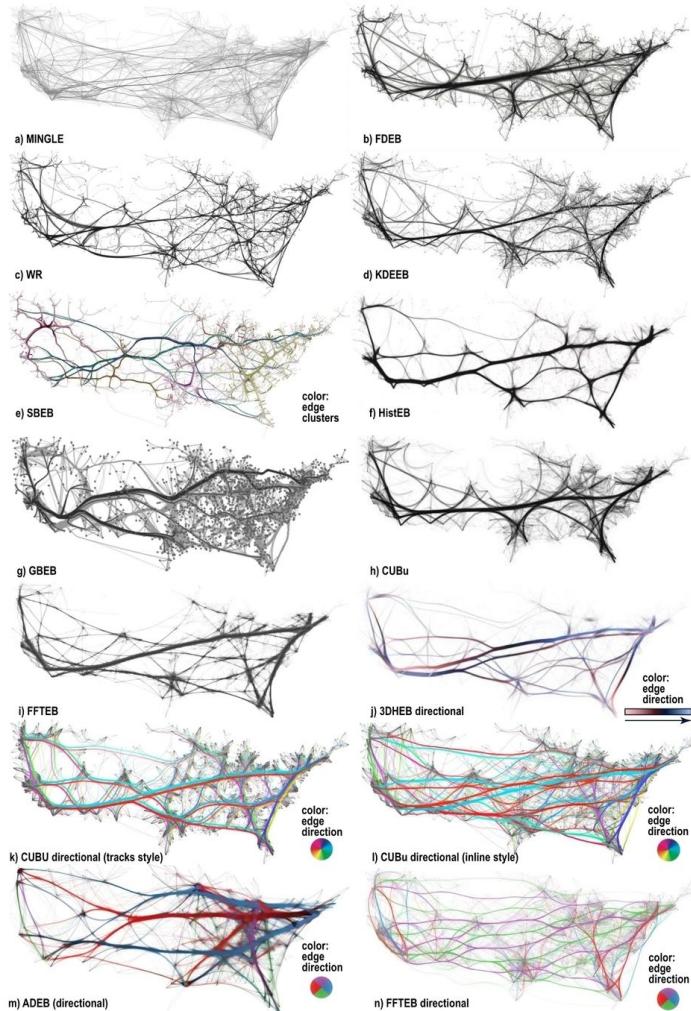
Les algorithmes de groupement des liens simplifient le dessin de réseaux

Mais ils produisent des résultats difficiles à contrôler

- Si l'objectif est de rendre le réseau plus beau, OK
- Si les trajectoires doivent avoir un sens, alors il faut les éviter
- Simplifier retire de l'information...

A. Lhuillier, C. Hurter, A. Telea.

**State of the Art in Edge and Trail Bundling Techniques**, in *Eurovis 2017 June. 2017*



# Trees

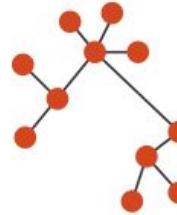
## Arrange Networks and Trees

### → Node–Link Diagrams

Connection Marks

NETWORKS

TREES

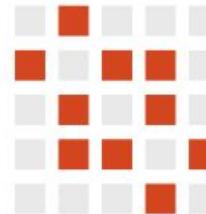


### → Adjacency Matrix

Derived Table

NETWORKS

TREES



### → Enclosure

Containment Marks

NETWORKS

TREES



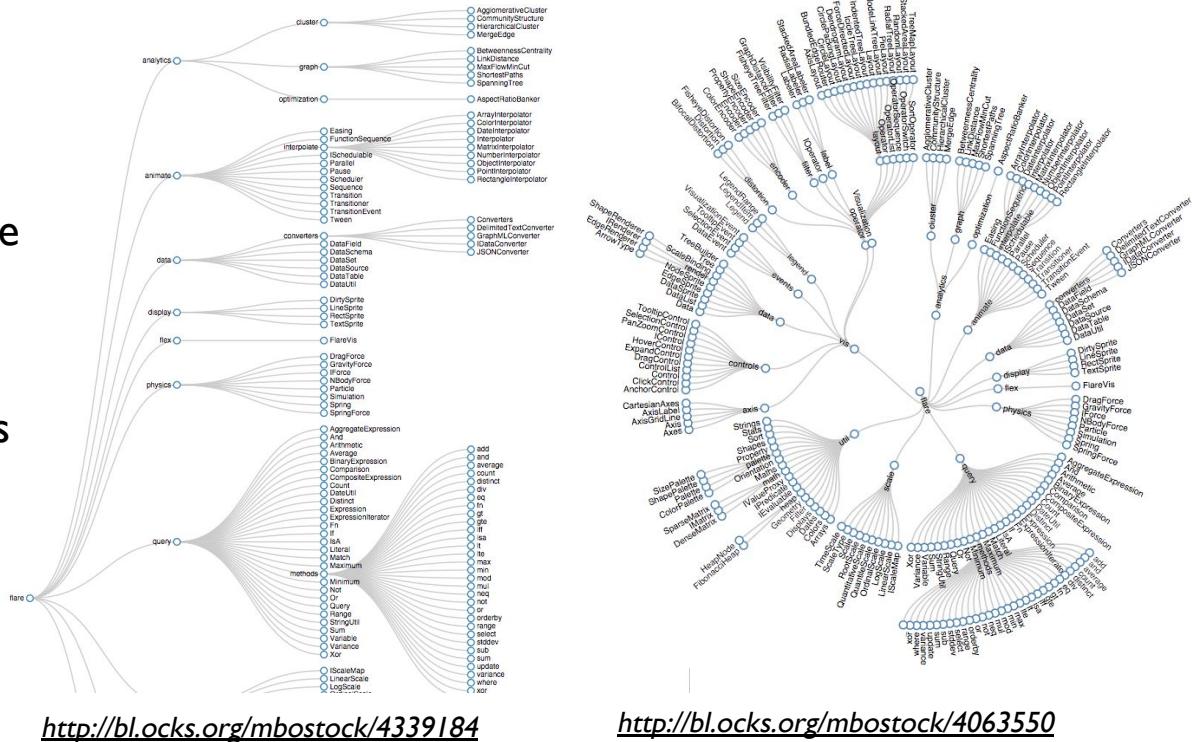
# Node-link trees

- Reingold-Tilford
  - tidy drawings of trees
    - exploit parent/child structure
  - allocate space: compact but without overlap
    - rectilinear and radial variants

[Tidier drawing of trees. Reingold and Tilford. IEEE Trans. Software Eng., SE-7(2):223–228, 1981.]

- nice algorithm writeup

<http://billmill.org/pymag-trees/>

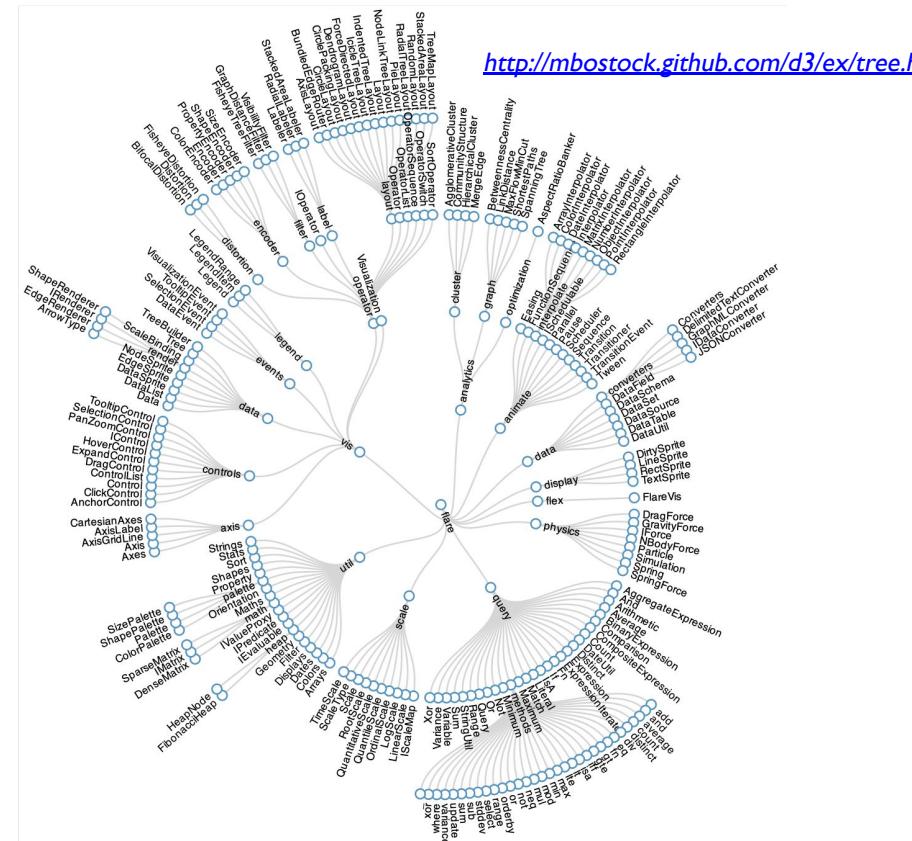


<http://bl.ocks.org/mbostock/4339184>

<http://bl.ocks.org/mbostock/4063550>

## Idiom: **radial node-link tree**

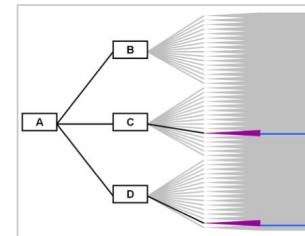
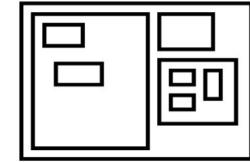
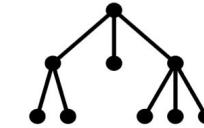
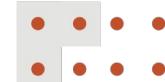
- data
    - tree
  - encoding
    - link connection marks
    - point node marks
    - radial axis orientation
      - angular proximity: siblings
      - distance from center: depth in tree
  - tasks
    - understanding topology, following paths
  - scalability
    - 1K - 10K nodes (with/without labels)



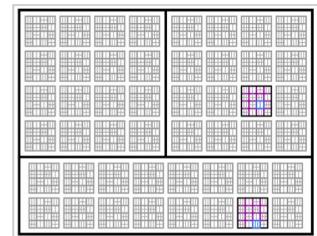
# Link marks: Connection and containment

- marks as links (vs. nodes)
  - common case in network drawing
  - 1D case: connection
    - ex: all node-link diagrams
    - emphasizes topology, path tracing
    - networks and trees
  - 2D case: containment
    - ex: all treemap variants
    - emphasizes attribute values at leaves (size coding)
    - only trees

→ Connection → Containment



Node-Link Diagram



Treemap

# Idiom: treemap

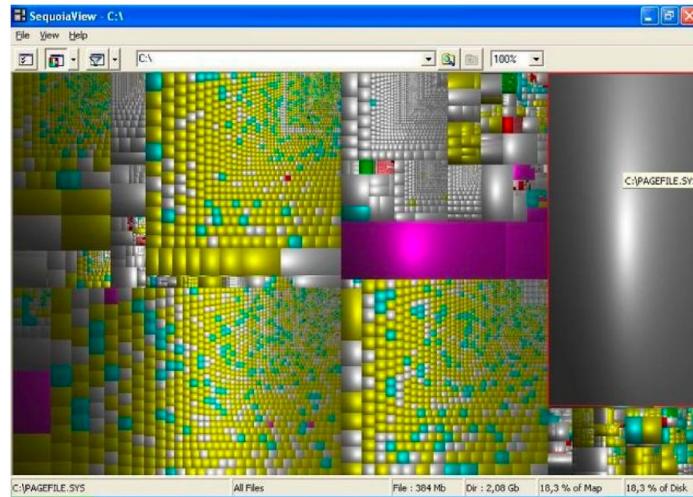
- data
  - tree
  - 1 quant attrib at leaf nodes
- encoding
  - area containment marks for hierarchical structure
  - rectilinear orientation
  - size encodes quant attrib
- tasks
  - query attribute at leaf nodes
  - ex: disk space usage within filesystem
- scalability
  - 1M leaf nodes

## → Enclosure

Containment Marks

NETWORKS

TREES



<https://www.win.tue.nl/sequoiaview/>

[Cushion Treemaps. van Wijk and van de Wetering.  
Proc. Symp. InfoVis 1999, 73-78.]

# Idiom: implicit tree layouts (sunburst, icicle plot)

- alternative to connection and containment: position
  - show parent-child relationships only through relative positions

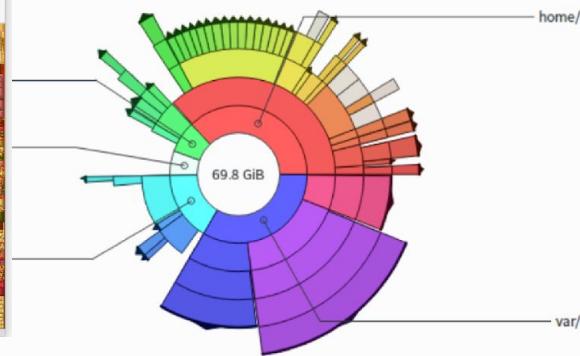
Treemap

containment



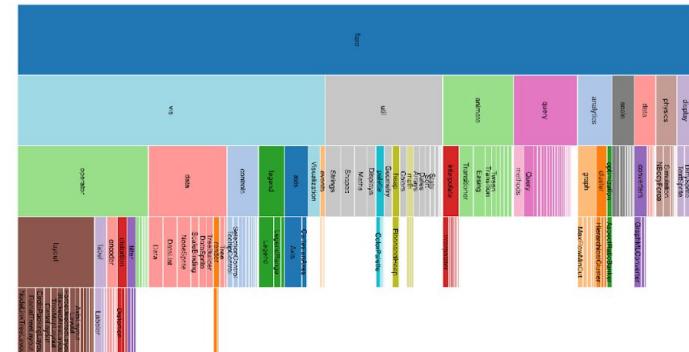
Sunburst

position (radial)



Icicle Plot

position (rectilinear)



# Idiom: implicit tree layouts (sunburst, icicle plot)

- alternative to connection and containment: position
  - show parent-child relationships only through relative positions

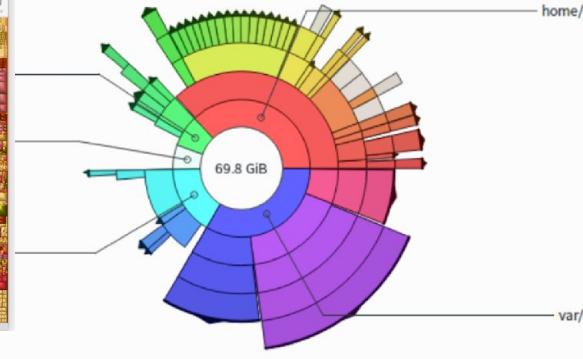
Treemap

containment  
only leaves visible



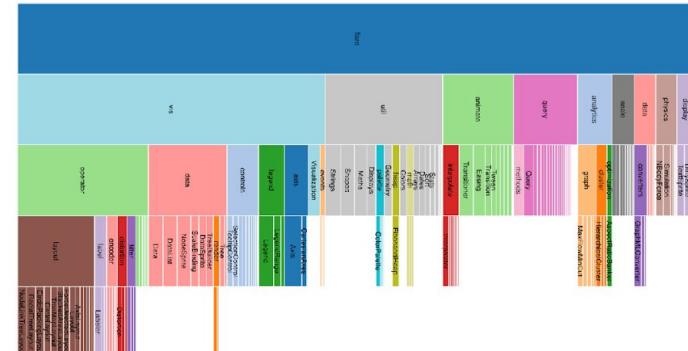
Sunburst

position (radial)  
inner nodes & leaves visible



Icicle Plot

position (rectilinear)  
inner nodes & leaves visible



# Idiom: implicit tree layouts (sunburst, icicle plot)

- alternative to connection and containment: position
  - show parent-child relationships only through relative positions

Treemap

containment  
only leaves visible



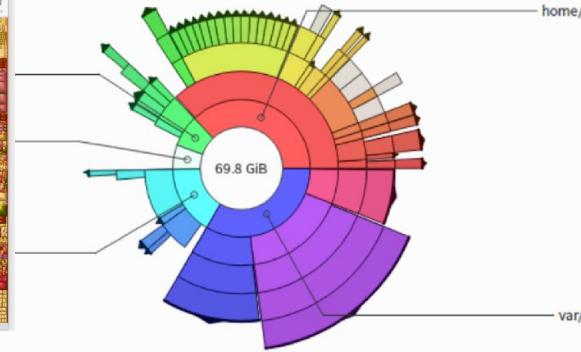
⇒ Implicit  
Spatial Position

✗ NETWORKS

✓ TREES

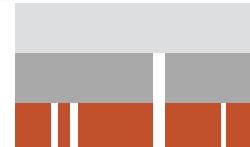
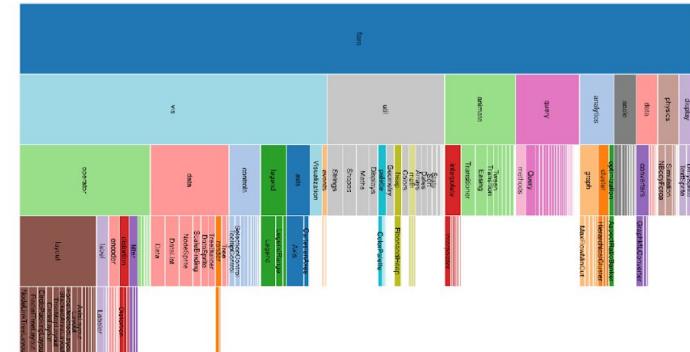
Sunburst

position (radial)  
inner nodes & leaves visible

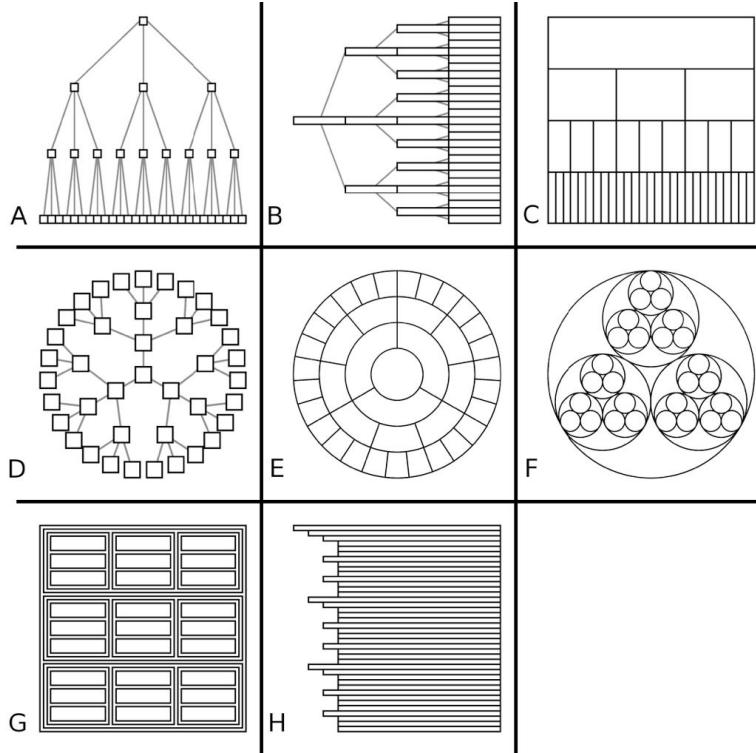


Icicle Plot

position (rectilinear)  
inner nodes & leaves visible



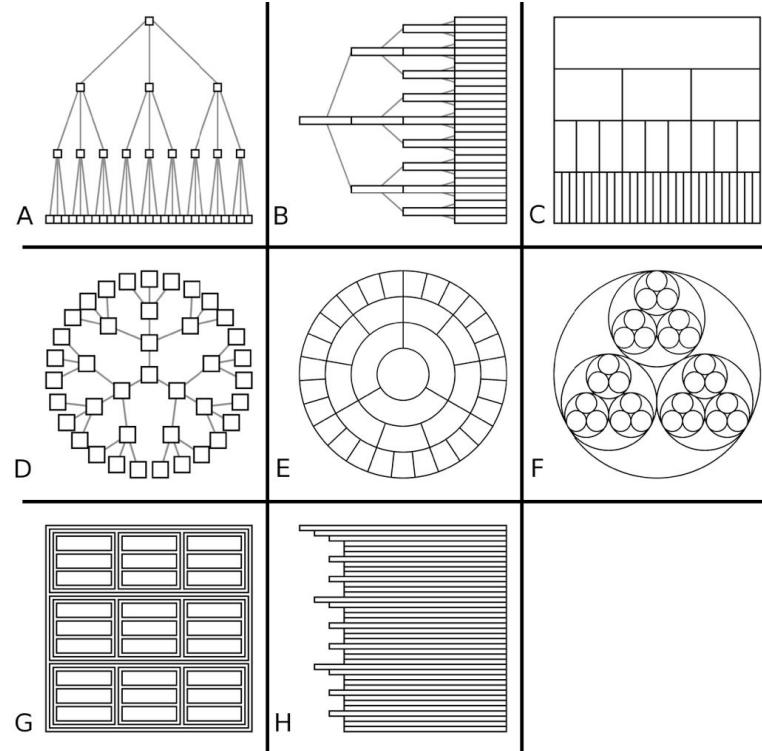
# Tree drawing idioms comparison



[Quantifying the Space-Efficiency of 2D Graphical Representations of Trees.  
McGuffin and Robert. *Information Visualization* 9:2 (2010), 115–140.]

# Comparison: tree drawing idioms

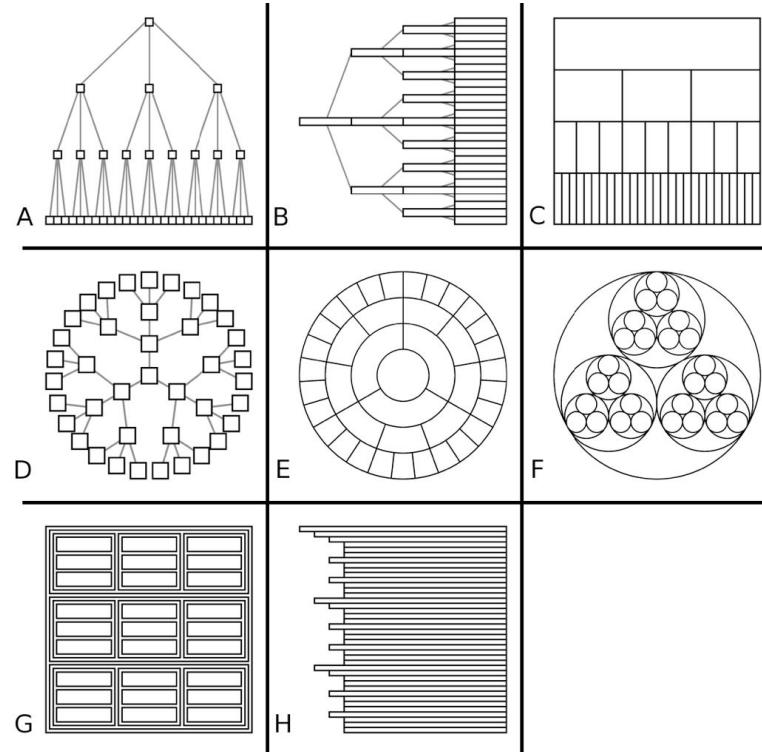
- data shown
  - link relationships
  - tree depth
  - sibling order



[Quantifying the Space-Efficiency of 2D Graphical Representations of Trees.  
McGuffin and Robert. *Information Visualization* 9:2 (2010), 115–140.]

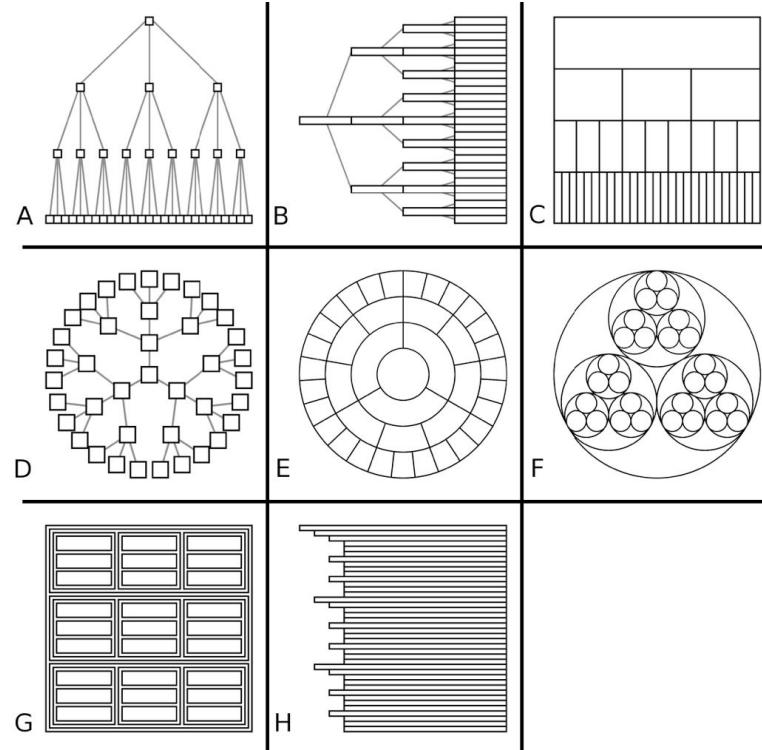
# Comparison: tree drawing idioms

- data shown
  - link relationships
  - tree depth
  - sibling order
- design choices
  - connection vs containment link marks
  - rectilinear vs radial layout
  - spatial position channels



# Comparison: tree drawing idioms

- data shown
  - link relationships
  - tree depth
  - sibling order
- design choices
  - connection vs containment link marks
  - rectilinear vs radial layout
  - spatial position channels
- considerations
  - redundant? arbitrary?
  - information density?
    - avoid wasting space
    - consider where to fit labels!



[Quantifying the Space-Efficiency of 2D Graphical Representations of Trees.  
McGuffin and Robert. *Information Visualization* 9:2 (2010), 115–140.]

# treevis.net: Many, many options!

How to cite this site?  
Check out other surveys!

treevis.net - A Visual Bibliography of Tree Visualization 2.0 by Hans-Jörg Schulz

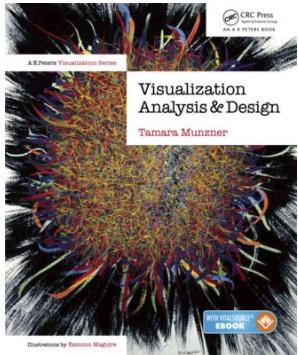
v.21-OCT-2014

Dimensionality      Representation      Alignment      Fulltext Search      Techniques Shown

All      All      All      277

The screenshot displays a collection of 120 thumbnail images arranged in a 10x12 grid, each illustrating a different tree visualization technique. The thumbnails include various types of hierarchical and network-like structures, such as sunburst charts, dendograms, phylogenetic trees, treemaps, and 3D visualizations. The images are diverse in color and complexity, demonstrating the wide range of applications for tree visualization.

<https://treevis.net/>



# Visualization Analysis & Design

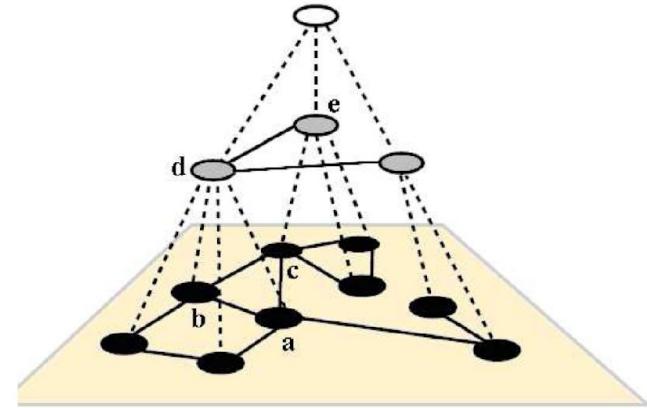
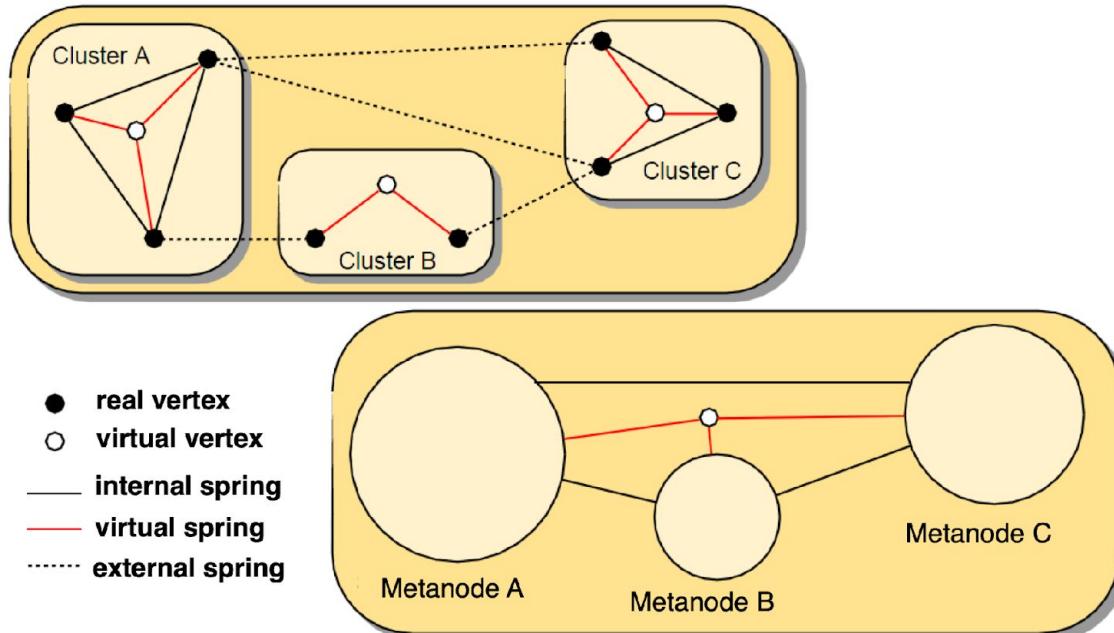
## *Network Data (Ch 9) II*

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# Multilevel networks

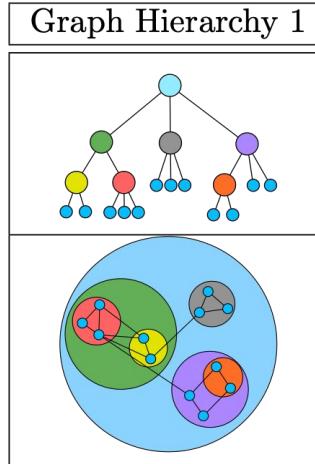
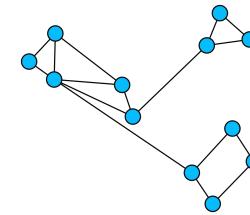
- derive cluster hierarchy of metanodes on top of original graph nodes



[Schulz 2004]

# Idiom: GrouseFlocks

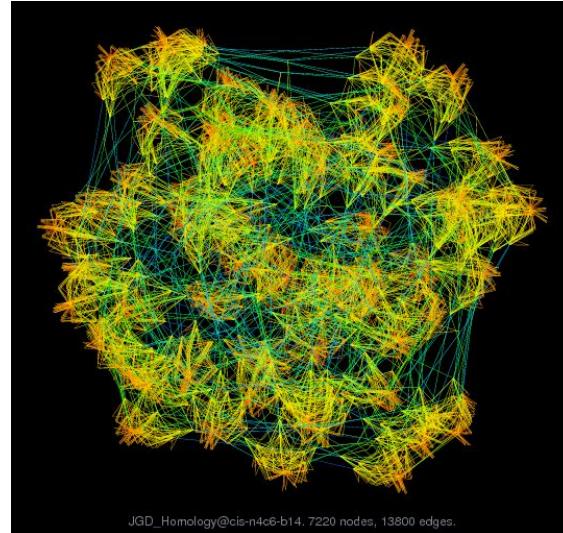
- data: compound network
  - network
  - cluster hierarchy atop it
    - derived or interactively chosen
- visual encoding
  - connection marks for network links
  - containment marks for hierarchy
  - point marks for nodes
- dynamic interaction
  - select individual metanodes in hierarchy to expand/contract



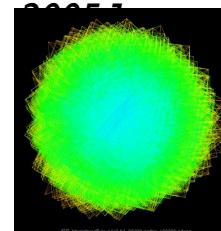
[[GrouseFlocks: Steerable Exploration of Graph Hierarchy Space](#). Archambault, Munzner, and Auber. IEEE TVCG 14(4):900-913, 2008.]

# Idiom: **sfdp** (multi-level force-directed placement)

- data: compound graph
  - original: network
  - derived: cluster hierarchy atop it
- considerations
  - better algorithm for same encoding technique
    - same: fundamental use of space
    - hierarchy used for algorithm speed/quality but not shown explicitly
- scalability
  - nodes, edges: 1K-10K
  - hairball problem eventually hits

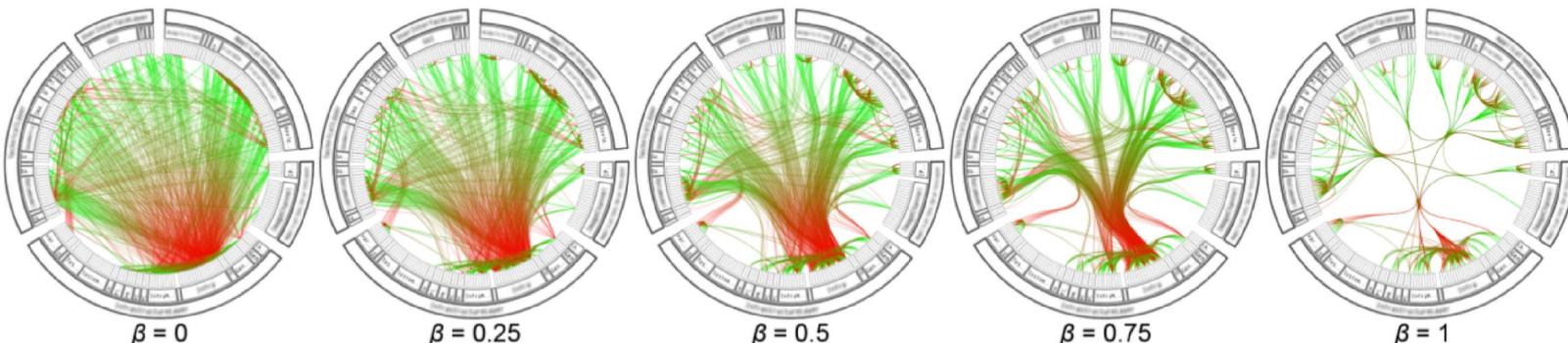


[Efficient and high quality force-directed graph drawing. Hu. The Mathematica Journal 10:37–71,



# Idiom: hierarchical edge bundling

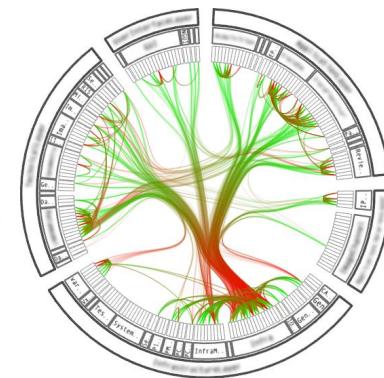
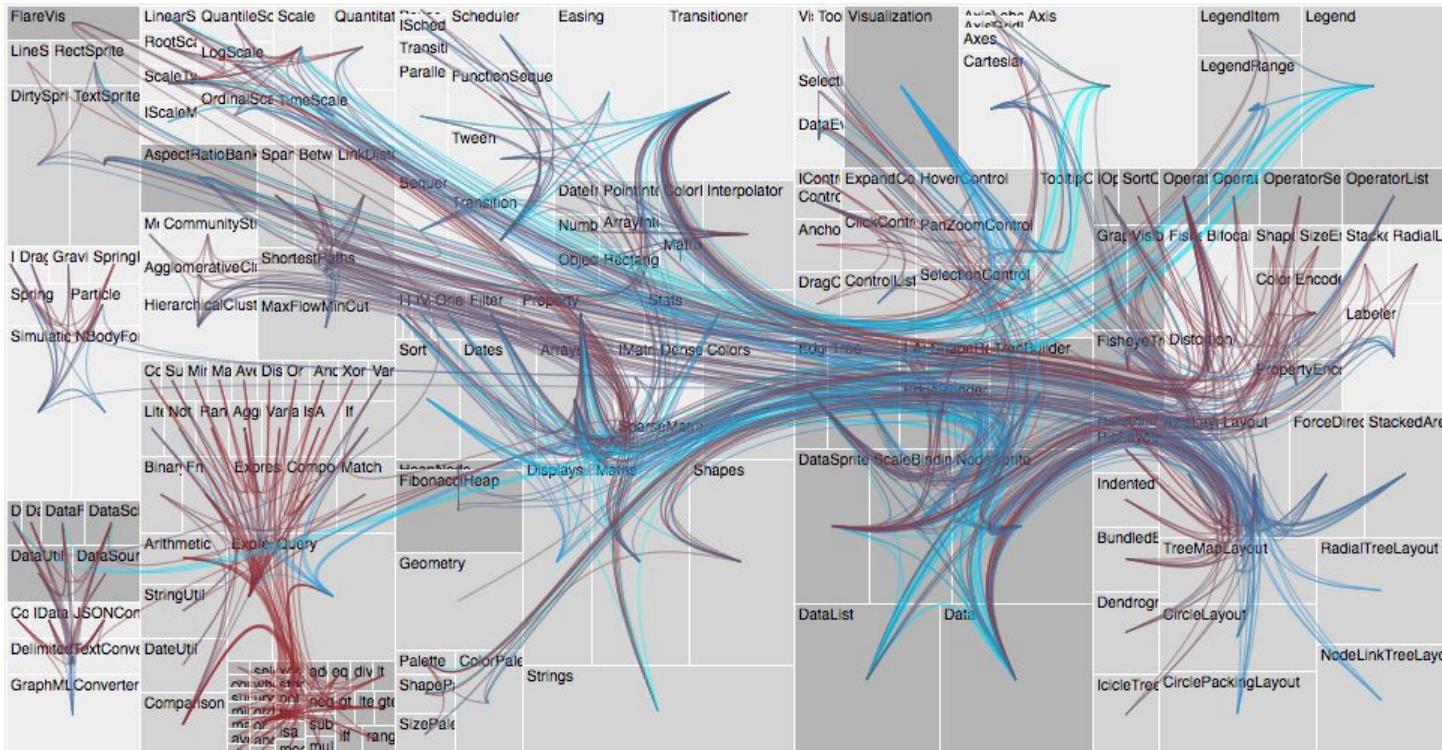
- data
  - any layout of compound network
    - network: software classes (nodes), import/export between classes (links)
    - cluster hierarchy: class package structure
  - derived: bundles of edges with same source/destination (multi-level)
- idiom: curve edge routes according to bundles
- task: edge clutter reduction



[Hierarchical Edge Bundles: Visualization of Adjacency Relations in Hierarchical Data. Danny Holten. TVCG 12(5):741-748 2006]

# Hierarchical edge bundling

- works for any layout: treemap vs radial



# Free Software

- Java
  - Cytoscape <https://cytoscape.org/>, Gephi <https://gephi.org/>
- C/C++/Python
  - GraphViz <https://www.graphviz.org/>, igraph <https://igraph.org/>, NetworkX <https://networkx.org/>
  - Tulip <https://tulip.labri.fr/site/>
- Web
  - D3 <https://d3js.org/>
  - Vega <https://vega.github.io/vega/examples/>
  - The Vistorian <https://vistorian.net/>
  - <https://demo.hedgedoc.org/features>
  - WebCola <https://ialab.it.monash.edu/webcola/>